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536. GRAPHIC APPROACH TO WEIGHTED CONJUNCTIVE AND DISJUNCTIVE MEANS CALCULATION*

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Graphic procedures for calculation of the weighted conjunctive and disjunctive means and weighted power means are considered. The displayed graphic procedures allow better insight into the properties of the considered class of means.

Let real numbers $y_1, y_2, a_1, a_2, \in [0, 1]$ be given. From Fig. 1 it follows $(y_1 - y_0)/a_2 = (y_0 - y_2)/a_1$, that is

$$y_0 = \frac{a_1}{a_1 + a_2} y_1 + \frac{a_2}{a_1 + a_2} y_2. \tag{1}$$

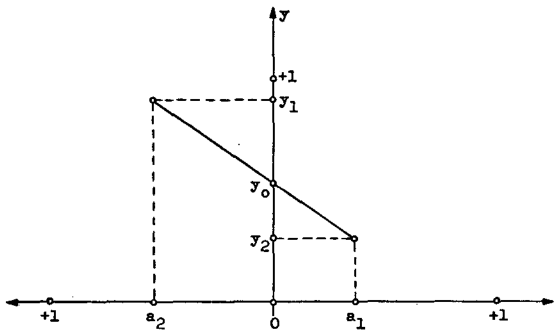

Fig. 1

Restrict the consideration to the case $a_1 + a_2 = 1$ and $y = f(x)$, where f is a strictly increasing function. In view of (1), it follows

$$x_0 = f^{-1}\left(a_1 f(x_1) + a_2 f(x_2)\right). \tag{2}$$

If the ordinate axis in Fig. 1 is scaled functionally and parametrized with the values of independent variable x, a nomogram for calculation of the values of functions of type (2) could be obtained. In the case when $f(x) = x^r$, it follows from (2) that the values of weighted power means for two variables [1]

$$x_0 = M_2^{[r]}(X_2;\, A_2), \quad X_n = (x_1, \ldots, x_n), \quad A_n = (a_1, \ldots, a_n), \quad \sum_{i=1}^{n} a_i = 1, \quad n > 1$$

may be calculated by the nomogram given in Fig. 1. Since

$$M_n^{[r]}(X_n;\, A_n) = \left\{(1 - a_n)\left[M_{n-1}^{[r]}(X_{n-1};\, B_{n-1})\right]^r + a_n x_n^r\right\}^{1/r}, \tag{3}$$

$$B_{n-1} = \left(\frac{a_1}{1 - a_n}, \ldots, \frac{a_{n-1}}{1 - a_n}\right),$$

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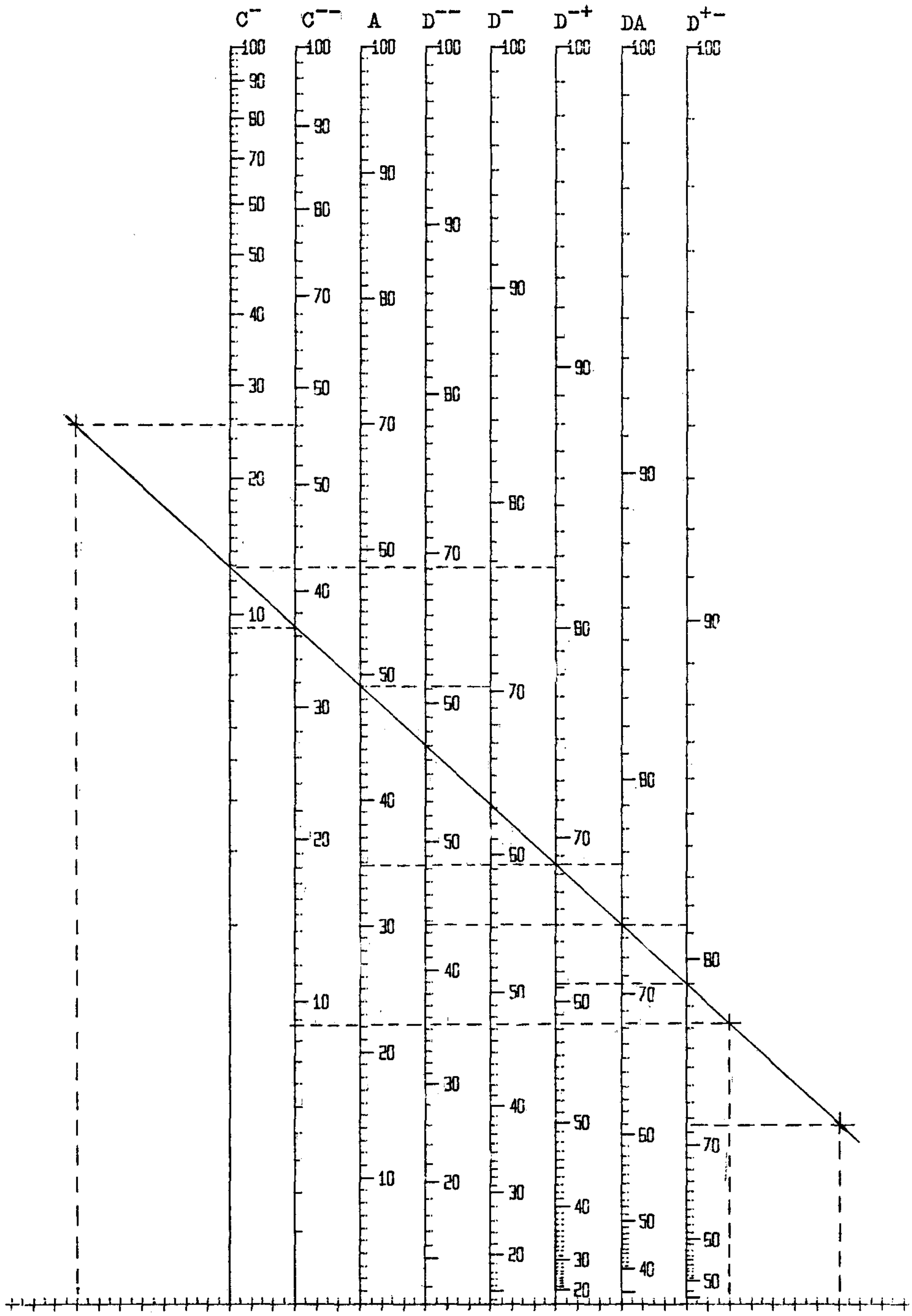


Fig. 2

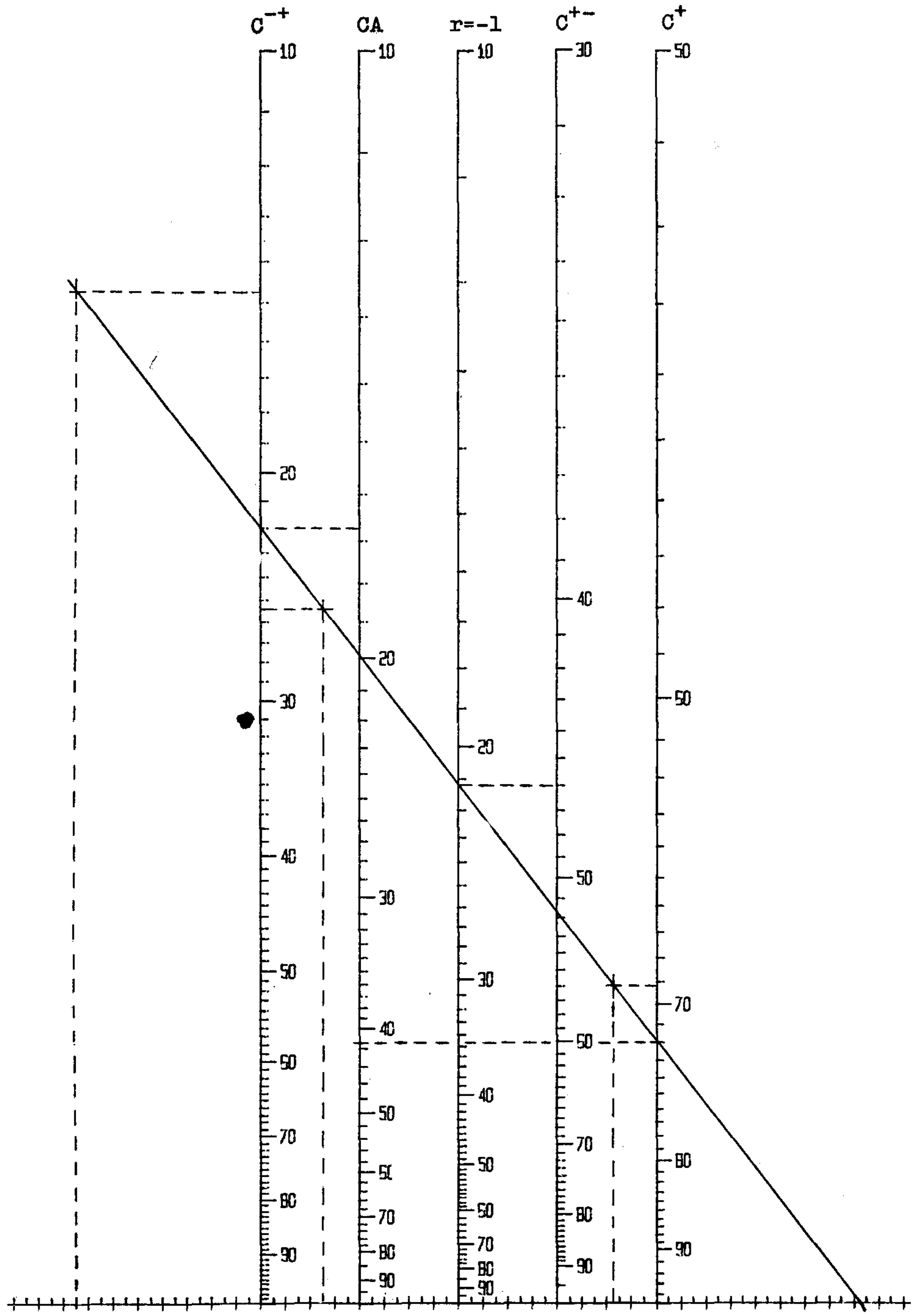

C-+
CA
r=-1
C+-
C+
10 20 30 40 50 60 70 80 90
10 20 30 40 50 60 70 80 90
10 20 30 40 50 60 70 80 90
30 40 50 60 70 80 90
50 60 70 80 90

Fig. 3

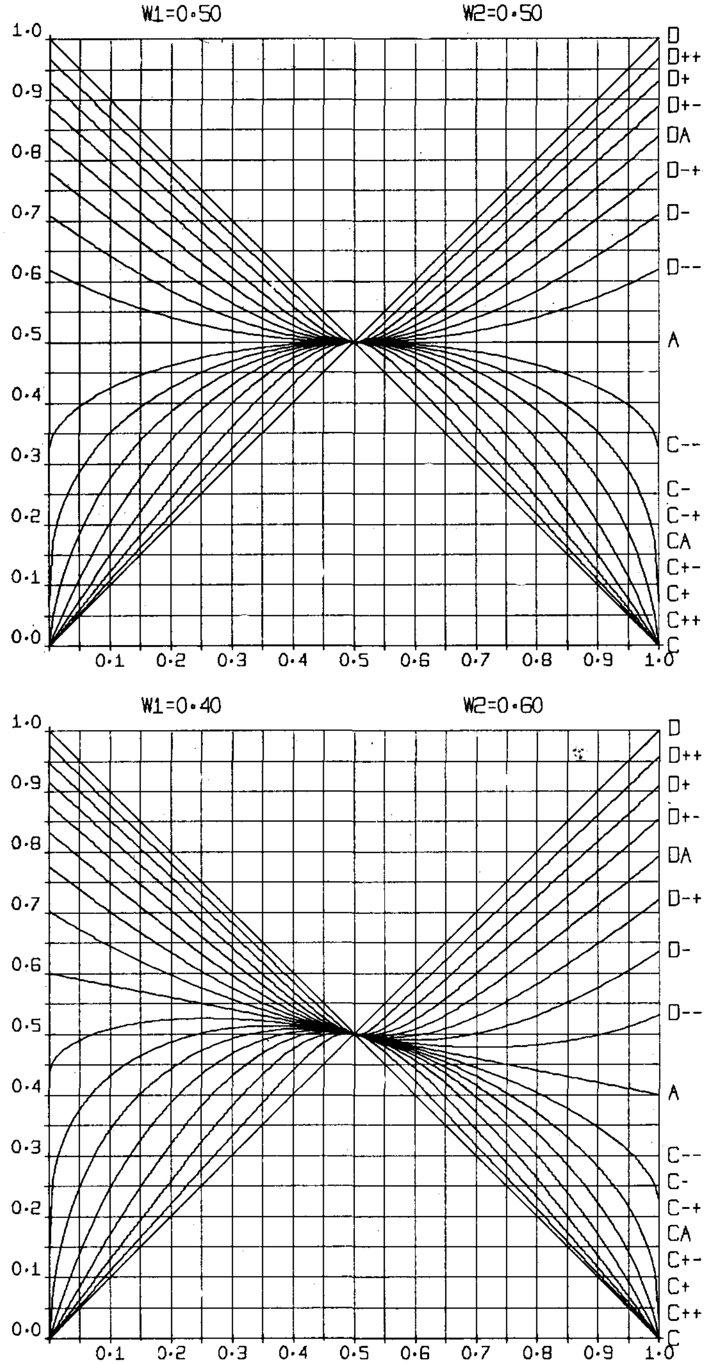
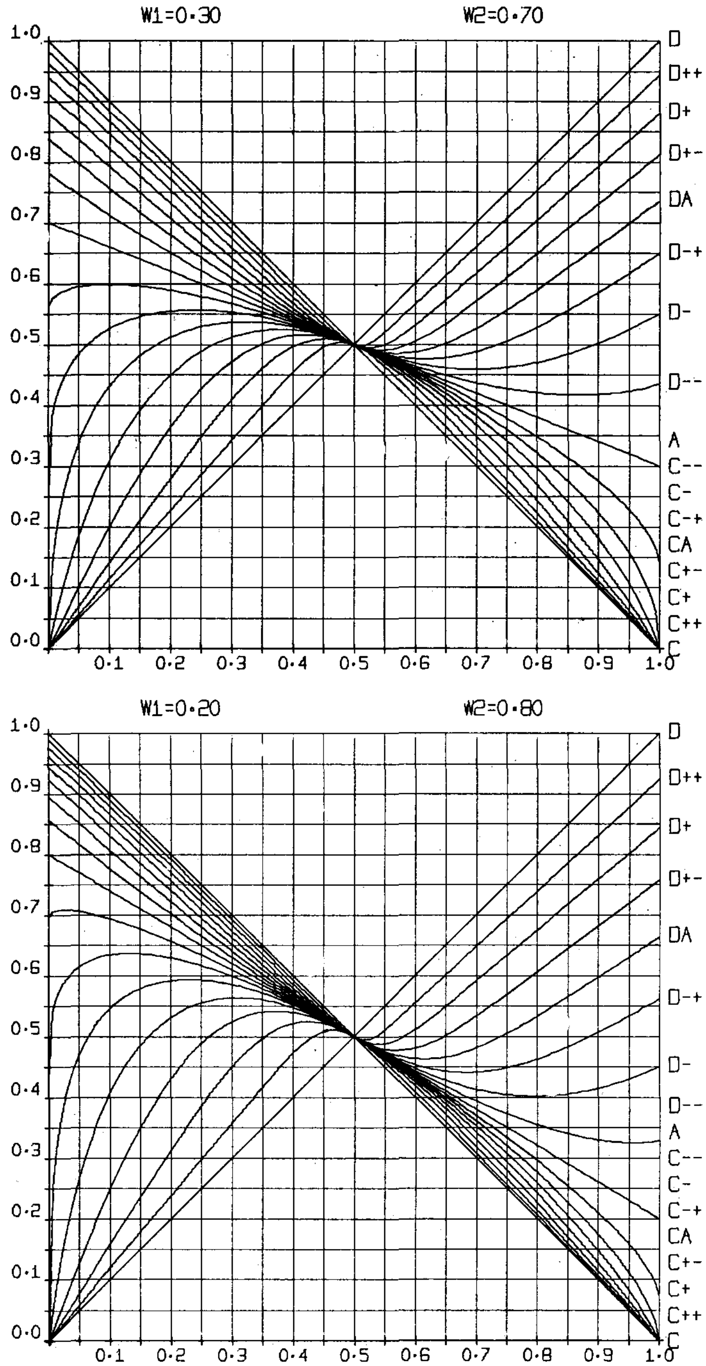

W1=0·50
W2=0·50
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0·0
0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0
D
D++
D+
D+-
DA
D-+
D-
D--
A
C--
C-
C-+
CA
C+-
C+
C++
C
W1=0·40
W2=0·60
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0·0
0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0
D
D++
D+
D+-
DA
D-+
D-
D--
A
C--
C-
C-+
CA
C+-
C+
C++
C

Fig. 4


W1=0·30
W2=0·70
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0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0·0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
D
D++
D+
D+-
DA
D-+
D-
D--
A
C--
C-
C-+
CA
C+-
C+
C++
C
W1=0·20
W2=0·80
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0·0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
D
D++
D+
D+-
DA
D-+
D-
D--
A
C--
C-
C-+
CA
C+-
C+
C++
C

Fig. 5

it follows that by successive applications of the nomogram given in Fig. 1, the values of weighted power means may be calculated even for cases with more than two variables. The given nomogram is especially suitable for calculation of the values of weighted conjunctive and disjunctive means [2] because the constant values of parameter r are used in this case. The corresponding function scales are shown in Figs. 2 and 3 together with the calculated means given as an illustration of the proposed procedure. The resultant means are presented in Table 1 (x_0 denotes the value read from the nomogram, and ε is the difference between the exact value and x_0).

Table 1

Operation	a_1	x_1	x_2	x_0	ε
C^-	0.3	0.255	0.093	0.129	0.00055
C^{--}	2/3	0.560	0.087	0.369	—0.00034
A	0.6	0.586	0.348	0.490	0.00080
D^{--}	0.5	0.691	0.437	0.570	0.00043
D^-	1/3	0.703	0.592	0.632	—0.00079
D^{-+}	2/7	0.825	0.612	0.686	—0.00057
DA	0.5	0.765	0.706	0.737	0.00023
D^{+-}	0.7	0.813	0.715	0.790	—0.00068
C^{-+}	0.25	0.147	0.254	0.220	0.00061
CA	0.75	0.167	0.414	0.199	0.00043
C^{+-}	0.5	0.460	0.600	0.516	0.00180
C^+	0.825	0.692	1.000	0.720	—0.00071

In the cases when the values to be averaged fall out the range of scales given in Figs. 2 and 3, one can proceed in the following way. Since

$$(4) \qquad M_2^{[r]}(X_2;\, W_2) = (x_1 + x_2)\, M_2^{[r]}(Z_2;\, W_2), \quad Z_2 = (z_1,\, z_2), \quad W_2 = (w_1,\, w_2),$$

$$z_1 + z_2 = w_1 + w_2 = 1, \quad z_1 = \frac{x_1}{x_1 + x_2}, \quad z_2 = \frac{x_2}{x_1 + x_2}, \quad x_1 + x_2 \neq 0,$$

the averaging of arbitrary values x_1 and x_2 may be reduced to averaging of normalized values z_1, z_2 by applying (4), and extended to greather number of variables by applying (3). The corresponding averaging functions are shown in Figs. 4 and 5 (the abscissa is z_1, and the ordinate is $M_2^{[r]}(Z_2;\, W_2)$).

REFERENCES

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2. J. J. Dujmović: *Weighted conjunctive and disjunctive means and their application in system evaluation.* These Publications № **461** — № **497** (1974), 147—158.