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357. A PROOF OF AN INEQUALITY OF H. THUNSDORFF*

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In his thesis, H. THUNSDORFF proved this result (see [**1**], p. 307):

Theorem. *Let m, n be non-negative numbers, $m \leqq n$ and let $f(x)$ be any nonnegative function convex in* $[0, 1]$ *satisfying* $f(0)=0$. *Then, this inequality holds*:

$$(1) \qquad \Big((m+1)\int_0^1 f(x)^m \, dx\Big)^{1/m} \leqq \Big((n+1)\int_0^1 f(x)^n \, dx\Big)^{1/n}.$$

A very simple proof of (1) will be offered.

Let $\varphi(x)$, $\psi(x)$ be functions, convex in $[0, 1]$ which satisfy these conditions: There exists an $x_0 \in [0, 1]$ so that for $x \leqq x_0$ $\varphi(x) \geqq \psi(x)$, for $x \geqq x_0$ $\varphi(x) \leqq \psi(x)$ holds and so that the area relation

$$(2) \qquad \int_0^{t_0} \big(\varphi(x)-\psi(x)\big) \, dx = \int_{t_0}^1 \big(\psi(x)-\varphi(x)\big) \, dx$$

is fulfilled. (2) implies that for any number $k \geqq 0$ the following moment inequality holds:

$$\int_0^1 \int_0^{\varphi(x)} y^k \, dy \, dx \leqq \int_0^1 \int_0^{\psi(x)} y^k \, dy \, dx$$

which in turn gives

$$(3) \qquad \int_0^1 \varphi(x)^{k+1} \, dx \leqq \int_0^1 \psi(x)^{k+1} \, dx.$$

Choosing $\varphi(x)=(c_m x)^m$ and $\psi(x)=f(x)^m$ we determine c_m so that (2) holds and obtain

$$(4) \qquad c_m = \Big((m+1)\int_0^1 f(x)^m \, dx\Big)^{1/m}.$$

* Presented June 2, 1971, by D. S. MITRINOVIĆ.

Finally, picking $k=\frac{n}{m}-1$ in (3) we get

$$c_m{}^n \int_0^1 x^n \, dx \leqq \int_0^1 f(x)^n \, dx.$$

This, together with (4), implies (1).

REFERENCE

1. D. S. MITRINOVIĆ: *Analytic inequalities.* New York — Heidelberg — Berlin, 1970.

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