NONLINEAR GENERALIZED CONTRACTIONS ON MENGER PM SPACES

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This paper presents a fixed point theorem for a self-mapping defined on probabilistic Menger spaces satisfying nonlinear generalized contractive type conditions. The theorem is an improvement of a result presented by B.S. CHOUDHURY, K. DAS: *A new contraction principle in Menger spaces,* Acta Math. Sin. (Engl. Ser.), **24** (2008), 1379–1386. This is illustrated with an example.

1. INTRODUCTION

The notion of probabilistic metric spaces, as a generalization of metric spaces, was introduced by K. MENGER [**9**] in 1942. SCHWEIZER and SKLAR [**12**] studied the properties of spaces introduced by K. Menger and gave some basic results on these spaces. They studied topology, convergence of sequences, continuity of mappings, defined the completeness of these spaces, etc.

Fixed point properties for mappings defined on probabilistic spaces were studied by many authors ([**1**], [**13**], [**6**], [**14**], [**11**]). Most of the properties which provide the existence of fixed point and common fixed point are of linear contractive type conditions.

The results in fixed point theory including nonlinear type contractive conditions were given by D. W. BOYD and J. S. W. WONG [**2**], S. JEŠIĆ et al. [**7**] and recently D. O'REGAN and R. SADAATI [**11**].

Altering distance functions in Menger PM-spaces have been recently considered by B. S. CHOUDHURY and K. DAS [**3**]. Some fixed point results involving altering distances in Menger PM-spaces were given by D. MIHEŢ in [**10**].

Generalized contractions of linear type on probabilistic metric spaces are introduced by ĆIRIĆ [**4**]. Many authors studied existence of fixed points for mappings satisfying generalized contractive type conditions, defined on various spaces [**5**]. We define nonlinear generalized contractive type condition involving altering distances in Menger PM-spaces. Also, we prove a fixed point result for mappings satisfying such type of conditions.

Many authors studied fixed point results considering different classes of t-norm [**3, 10**]. We consider t-norm which satisfies $T(a, a) \geq a$.

2. PRELIMINARIES

In the standard notation, let $D+$ be the set of all distribution functions $F : \mathbb{R} \to [0, 1]$, such that F is a nondecreasing, left-continuous mapping, which satisfies $F(0) = 0$ and $\sup_{x \in \mathbb{R}} F(x) = 1$. The space $D+$ is partially ordered by the usual point-wise ordering of functions, i.e. $F \leq G$ if and only if $F(t) \leq G(t)$ for all $t \in \mathbb{R}$. The maximal element for $D+$ in this order is the distribution function given by

$$\varepsilon_0(t) = \left\{ \begin{array}{ll} 0, & t \leq 0, \\ 1, & t > 0. \end{array} \right.$$

Definition 2.1. ([**12**]) *A binary operation* $T : [0, 1] \times [0, 1] \to [0, 1]$ *is continuous t-norm if T satisfies the following conditions*:

(a) *T is commutative and associative*;
(b) *T is continuous*;
(c) *$T(a, 1) = a$ for all $a \in [0, 1]$*;
(d) *$T(a, b) \leq T(c, d)$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.*

Examples of t-norm are $T(a, b) = \min\{a, b\}$ and $T(a, b) = ab$.
The t-norms are defined recursevly by $T^1 = T$ and

$$T^n(x_1, \ldots, x_{n+1}) = T(T^{n-1}(x_1, \ldots, x_n), x_{n+1}).$$

for $n \geq 2$ and $x_i \in [0, 1]$ for all $i \in \{1, \ldots, n + 1\}$.

Definition 2.2. *A Menger probabilistic metric space* (*briefly, Menger PM-space*) *is a triple $(X, \mathcal{F}, T)$ where X is a nonempty set, T is a continuous t-norm, and $\mathcal{F}$ is a mapping from $X \times X$ into D^+ such that, if $F_{x,y}$ denotes the value of $\mathcal{F}$ at the pair (x, y), the following conditions hold*:

(PM1)   *$F_{x,y}(t) = \varepsilon_0(t)$ if and only if $x = y$*;
(PM2)   *$F_{x,y}(t) = F_{y,x}(t)$*;
(PM3)   *$F_{x,z}(t + s) \geq T(F_{x,y}(t), F_{y,z}(s))$ for all $x, y, z \in X$ and $s, t \geq 0$.*

REMARK 2.3. ([**13**]) Every metric space is a PM-space. Let (X, d) be a metric space and $T(a, b) = \min\{a, b\}$ is a continuous t-norm. Define $F_{x,y}(t) = \varepsilon_0(t - d(x, y))$ for all $x, y \in X$ and $t > 0$. The triple $(X, \mathcal{F}, T)$ is a PM-space induced by the metric d.

Definition 2.4. *Let $(X, \mathcal{F}, T)$ be a Menger PM-space.*

(1) A sequence $\{x_n\}_n$ in X is said to be convergent to x in X if, for every $\varepsilon > 0$ and $\lambda > 0$ there exists positive integer N such that $F_{x_n,x}(\varepsilon) > 1 - \lambda$ whenever $n \geq N$.

(2) A sequence $\{x_n\}_n$ in X is called Cauchy sequence if, for every $\varepsilon > 0$ and $\lambda > 0$ there exists positive integer N such that $F_{x_n,x_m}(\varepsilon) > 1 - \lambda$ whenever $n, m \geq N$.

(3) A Menger PM-space is said to be complete if every Cauchy sequence in X is convergent to a point in X.

The (ε, λ)-topology ([**12**]) in Menger PM-space $(X, \mathcal{F}, T)$ is introduced by the family of neighbourhoods $\mathcal{N}_x$ of a point $x \in X$ given by

$$\mathcal{N}_x = \{N_x(\varepsilon, \lambda) : \ \varepsilon > 0, \lambda \in (0, 1)\}$$

where

$$N_x(\varepsilon, \lambda) = \{y \in X : \ F_{x,y}(\varepsilon) > 1 - \lambda\}.$$

The (ε, λ)-topology is a Hausdorff topology. In this topology the function f is continuous in $x_0 \in X$ if and only if for every sequence $x_n \to x_0$ it holds that $f(x_n) \to f(x_0)$.

The following lemma is proved by B. SCHWEIZER and A. SKLAR.

Lemma 2.5. ([**12**]) *Let $(X, \mathcal{F}, T)$ be a Menger PM-space. Then the function $\mathcal{F}$ is lower semi-continuous for every fixed $t > 0$, i.e. for every fixed $t > 0$ and every two convergent sequences $\{x_n\}, \{y_n\} \subseteq X$ such that $x_n \to x, y_n \to y$ it follows that*

$$\liminf_{n \to \infty} F_{x_n,y_n}(t) = F_{x,y}(t).$$

KHAN et al. in [**8**] introduced the concept of altering distance functions that alter the distance between two points in metric spaces.

Definition 2.6. ([**8**]) *A function $h : [0, \infty) \to [0, \infty)$ is an altering distance function if*

  (i) h is monotone increasing and continuous and
  (ii) $h(t) = 0$ if and only if $t = 0$.

B. S. CHOUDHURY and K. DAS [**3**] extended the concept of altering distance functions to Menger PM-spaces.

Definition 2.7. ([**3**]) *A function $\phi : [0, \infty) \to [0, \infty)$ is said to be a Φ-function if the following conditions hold:*

  (i) $\phi(t) = 0$ if and only if $t = 0$;
  (ii) ϕ is strictly increasing and $\phi(t) \to \infty$ as $t \to \infty$;
  (iii) ϕ is left-continuous in $(0, \infty)$;
  (iv) ϕ is continuous at 0.

The class of all Φ-functions will be denoted by Φ.

Theorem 2.8. ([**3**]) *Let* $(X, \mathcal{F}, T_M)$ *be a complete Menger PM-space, with contin-uous t-norm* T_M *given by* $T_M(a, b) = \min\{a, b\}$ *and* f *be a continuous self-mapping on* X *such that for every* $x, y \in X$, *and all* $t > 0$ *holds*

$$(1) \qquad\qquad\qquad F_{fx,fy}(\phi(t)) \geq F_{x,y}(\phi(t/c))$$

where ϕ *is a* Φ*-function and* $0 < c < 1$. *Then* f *has a unique fixed point.*

Lemma 2.9. *Let* $(X, \mathcal{F}, T)$ *be a Menger PM-space. Let* $\phi : [0, \infty) \to [0, \infty)$ *be a* Φ*-function. Then the following statement holds.*

> *If for* $x, y \in X, 0 < c < 1$, *we have* $F_{x,y}(\phi(t)) \geq F_{x,y}(\phi(t/c))$ *for all* $t > 0$ *then* $x = y$.

Proof. From the fact that ϕ is strictly increasing, and since $0 < c < 1$, by induction we get $F_{x,y}(\phi(t)) \geq F_{x,y}(\phi(t/c)) \geq \cdots \geq F_{x,y}(\phi(t/c^n))$ Taking $\liminf$ as $n \to \infty$ we get $F_{x,y}(\phi(t)) \geq 1$, i. e. $x = y$.

3. MAIN RESULTS

The motivation for this paper is provided in [**4**] where Ćirić introduced a notion of generalized contraction on PM-spaces of linear type and proved a fixed point theorem for generalized contraction f defined on f-orbitally complete Menger PM-space with continuous t-norm T which satisfies $T(a, a) \geq a$ for every $a \in [0, 1]$.

In this paper we will improve this result by introducing the generalized con-traction of nonlinear type on PM-spaces with t-norm which satisfies $T(a, a) \geq a$ for every $a \in [0, 1]$.

Theorem 3.1. *Let* $(X, \mathcal{F}, T)$ *be a complete Menger PM-space with continuous t-norm* T *which satisfies* $T(a, a) \geq a$ *for every* $a \in [0, 1]$. *Let* $c \in (0, 1)$ *be fixed. If for a* Φ*-function* ϕ *and a self-mapping* f *on* X *holds*

$$(2) \qquad F_{fx,fy}(\phi(t)) \geq \min\left\{F_{x,y}(\phi(t/c)), F_{x,fx}(\phi(t/c)), F_{y,fy}(\phi(t/c)),\right.$$

$$\left. F_{x,fy}(2\phi(t/c)), F_{y,fx}(2\phi(t/c))\right\}$$

for every $x, y \in X$ *and all* $t > 0$, *then* f *has a unique fixed point in* X.

Proof. First note that for every t-norm T which satisfies $T(a, a) \geq a$, for every $a, b \in [0, 1]$ it holds $T(a, b) \geq T(\min\{a, b\}, \min\{a, b\}) \geq \min\{a, b\}$. From previ-ous, property (PM3) and the fact that T is nondecreasing we have that for every $x, y, z \in X$ and all $t > 0$ holds

$$(3) \qquad\qquad\qquad F_{x,y}(2t) \geq \min\{F_{x,z}(t), F_{y,z}(t)\}.$$

We shall prove that from previous inequalities follows that

$$(4) \qquad F_{fx,fy}(\phi(t)) \geq \min\{F_{x,y}(\phi(t/c)), F_{x,fx}(\phi(t/c)), F_{y,fy}(\phi(t/c))\}$$

holds for every $x, y \in X$ and all $t > 0$.

From the property of t-norm and (2) we have that following inequalities hold.

$$
\begin{aligned}
F_{fx,fy}&(\phi(t)) \\
&\geq \min \left\{ F_{x,y}(\phi(t/c)), F_{x,fx}(\phi(t/c)), F_{y,fy}(\phi(t/c)), F_{x,fy}(2\phi(t/c)), F_{y,fx}(2\phi(t/c)) \right\} \\
&\geq \min \left\{ F_{x,y}(\phi(t/c)), F_{x,fx}(\phi(t/c)), F_{y,fy}(\phi(t/c)), \right. \\
&\qquad \left. T(F_{x,fx}(\phi(t/c)), F_{fx,fy}(\phi(t/c))), T(F_{y,fy}(\phi(t/c)), F_{fy,fx}(\phi(t/c))) \right\} \\
&\geq \min \left\{ F_{x,y}(\phi(t/c)), F_{x,fx}(\phi(t/c)), F_{y,fy}(\phi(t/c)), \right. \\
&\qquad \left. \min\{F_{x,fx}(\phi(t/c)), F_{fx,fy}(\phi(t/c))\}, \min\{F_{y,fy}(\phi(t/c)), F_{fy,fx}(\phi(t/c))\} \right\} \\
&\geq \min\{F_{x,y}(\phi(t/c)), F_{x,fx}(\phi(t/c)), F_{y,fy}(\phi(t/c)), F_{fx,fy}(\phi(t/c))\}.
\end{aligned}
$$

If we assume that $F_{fx,fy}(\phi(t/c))$ is the minimum, then from Lemma 2.9 it holds that $fx = fy$, i.e. $F_{fx,fy}(\phi(t)) = 1$ and it follows that inequality (4) holds.

If we assume that the element $F_{fx,fy}(\phi(t/c))$ is not the minimum, then inequality (4) holds.

Now let $x_0 \in X$ be an arbitrary point. Let us define a sequence by $x_n = fx_{n-1}$. We will show that $\{x_n\}$ is a Cauchy sequence.

Let $t > 0$ and $\varepsilon \in (0, 1)$. From properties (i) and (iv) of Φ-function it follows that there exists $r > 0$ such that $t > \phi(r)$. Then for $n \in \mathbb{N}$ we get that

$$
\begin{aligned}
F_{x_n,x_{n+1}}(t) &\geq F_{x_n,x_{n+1}}(\phi(r)) = F_{fx_{n-1},fx_n}(\phi(r)) \\
&\geq \min\{F_{x_{n-1},x_n}(\phi(r/c)), F_{x_{n-1},fx_{n-1}}(\phi(r/c)), F_{x_n,fx_n}(\phi(r/c))\} \\
&= \min\{F_{x_{n-1},x_n}(\phi(r/c)), F_{x_n,x_{n+1}}(\phi(r/c))\}.
\end{aligned}
$$

We shall prove that

$$
(5) \qquad\qquad F_{x_n,x_{n+1}}(\phi(r)) \geq F_{x_{n-1},x_n}(\phi(r/c))
$$

holds.

If we assume that $F_{x_n,x_{n+1}}(\phi(r/c))$ is the minimum, then from Lemma 2.9 it holds that $F_{x_n,x_{n+1}}(\phi(r)) = 1$, i. e. inequality (5) holds.

If we assume that $F_{x_n,x_{n+1}}(\phi(r/c))$ is not the minimum, then the minimum is $F_{x_{n-1},x_n}(\phi(r/c))$, i.e. inequality (5) holds.
Since ϕ is strictly increasing, we have

$$
F_{x_n,x_{n+1}}(t) \geq F_{x_n,x_{n+1}}(\phi(r)) \geq F_{x_{n-1},x_n}(\phi(r/c)) \geq \cdots \geq F_{x_0,x_1}(\phi(r/c^n)),
$$

i.e.

$$
(6) \qquad\qquad F_{x_n,x_{n+1}}(t) \geq F_{x_0,x_1}(\phi(r/c^n))
$$

for arbitrary $n \in \mathbb{N}$.

Let $m, n \in \mathbb{N}$, we can assume that $m \geq n$. From (3), by induction we get

$$
F_{x_n,x_m}((m-n)t) \geq \min\{F_{x_n,x_{n+1}}(t), \ldots F_{x_{m-1},x_m}(t)\}.
$$

From previous and (6) we get

$$F_{x_n,x_m}((m-n)t) \geq \min\{F_{x_0,x_1}(\phi(r/c^n)), \ldots, F_{x_0,x_1}(\phi(r/c^{m-1})).$$

From the fact that F_{x_0,x_1} is non-decreasing and Φ-function is strictly increasing, the minimum of the right hand side is $F_{x_0,x_1}(\phi(r/c^n))$, i.e.

$$F_{x_n,x_m}((m-n)t) \geq F_{x_0,x_1}(\phi(r/c^n)).$$

Since ϕ is strictly increasing and $\liminf_{n\to\infty} \phi(t) = \infty$, there exists $n_0 \in \mathbb{N}$ such that $F_{x_0,x_1}(\phi(r/c^n)) > 1 - \varepsilon$, whenever $n \geq n_0$. From the previous it follows that for every $m \geq n \geq n_0$ holds

(7) $$F_{x_n,x_m}((m-n)t) \geq 1 - \varepsilon.$$

Since $t > 0$ and $\varepsilon \in (0,1)$ are arbitrary, we have that $\{x_n\}$ is Cauchy sequence in complete Menger PM-space, thus there exists $z \in X$ such that $z = \lim_{n\to\infty} x_n$. We will show that z is a fixed point of f.

From properties of t-norm T and Φ-function ϕ we have that for every $x, y \in X$ and all $t > 0$ there exist $r > 0$ such that $t > \phi(r)$ and $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ holds

$$F_{fz,z}(t) \geq T\left(F_{fz,x_n}(\phi(r)), F_{x_n,z}(t - \phi(r))\right) \geq \min\{F_{fz,x_n}(\phi(r)), F_{x_n,z}(t - \phi(r))\}.$$

Since $z = \lim_{n\to\infty} x_n$, for arbitrary $\varepsilon \in (0,1)$ holds $F_{x_n,z}(t - \phi(r)) > 1 - \varepsilon$.

Hence we get that the following holds

$$F_{fz,z}(t) \geq \min\{F_{fz,x_n}(\phi(r)), 1 - \varepsilon\}.$$

Since $\varepsilon > 0$ is arbitrary, we have that $F_{fz,z}(t) \geq F_{fz,x_n}(\phi(r))$.

From the definition of $\{x_n\}$, (4) and (7) we have that

$$\begin{aligned}
F_{fz,z}(t) &\geq F_{fz,x_n}(\phi(r)) = F_{fz,fx_{n-1}}(\phi(r)) \\
&\geq \min\{F_{z,x_{n-1}}(\phi(r/c)), F_{z,fz}(\phi(r/c)), F_{x_{n-1},fx_{n-1}}(\phi(r/c))\} \\
&= \min\{F_{z,x_{n-1}}(\phi(r/c)), F_{z,fz}(\phi(r/c)), F_{x_{n-1},x_n}(\phi(r/c))\} \\
&\geq \min\{1 - \varepsilon, F_{z,fz}(\phi(r/c)), 1 - \varepsilon\}.
\end{aligned}$$

Since $\varepsilon \in (0,1)$ is arbitrary, we have that

$$F_{fz,x_n}(\phi(r)) \geq F_{z,fz}(\phi(r/c)).$$

Taking $\liminf$ as $n \to \infty$ we get

$$F_{fz,z}(\phi(r)) \geq F_{z,fz}(\phi(r/c)),$$

and applying Lemma 2.9 we have that $z = fz$, i.e. z is a fixed point of f.

Let us prove that z is a unique fixed point of f. Let $y \in X$ be another fixed point of f, i.e. $f(y) = y$. For all $t > 0$ there exists $r > 0$ such that $t > \phi(r)$ and from (4) it follows that the following holds

$$\begin{aligned}
F_{z,y}(t) &\geq F_{z,y}(\phi(r)) = F_{fz,fy}(\phi(r)) \\
&\geq \min\{F_{z,y}(\phi(r/c)), F_{z,fz}(\phi(r/c)), F_{y,fy}(\phi(r/c))\} \\
&= \min\{F_{z,y}(\phi(r/c)), F_{z,z}(\phi(r/c)), F_{y,y}(\phi(r/c))\} \\
&= \min\{F_{z,y}(\phi(r/c)), 1, 1\} = F_{z,y}(\phi(r/c)).
\end{aligned}$$

From Lemma 2.9 it follows that $y = z$, i.e. z is a unique fixed point of f.

It is clear that the result presented in Theorem 3.1 is an improvement of result given by B.S. Choudhury and K. Das in [3] here stated in Theorem 2.8.

REMARK 3.2. Theorem 2.8 is a consequence of Theorem 3.1.

EXAMPLE 3.3. Let $(X, \mathcal{F}, T)$ be a complete Menger PM-space induced by a metric $d(x, y) = |x - y|$ on $X = [0, 1] \subset \mathbb{R}$ given in Remark 2.3. Let $\phi(t) = t$, for all $t > 0$, $c = \dfrac{1}{2}$ and

$$f(x) = \begin{cases} \dfrac{x}{4}, & x \in \left[0, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, 1\right] \\[2mm] 0, & x = \dfrac{1}{2}. \end{cases}$$

Note that ϕ is a Φ-function.

We shall prove that the condition (2) of Theorem 3.1 is satisfied. We will consider three possibilities.

If $x, y \in \left[0, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, 1\right]$ we get

$$F_{fx,fy}(\phi(t)) = \varepsilon_0\left(t - \left|\dfrac{x}{4} - \dfrac{y}{4}\right|\right) = \varepsilon_0(4t - |x - y|) \geq \varepsilon_0(2t - |x - y|) = F_{x,y}(\phi(2t)),$$

thus the condition (2) is satisfied.

If $x \in \left[0, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, 1\right]$ and $y = \dfrac{1}{2}$ we get

$$\begin{aligned}
F_{fx,fy}(\phi(t)) &= \varepsilon_0\left(t - \left|\dfrac{x}{4} - 0\right|\right) = \varepsilon_0(4t - x) \\
&\geq \varepsilon_0(4t - 1) = \varepsilon_0\left(2t - \left|\dfrac{1}{2} - 0\right|\right) = \varepsilon_0(2t - |y - fy|) = F_{y,fy}(\phi(2t)),
\end{aligned}$$

thus the condition (2) is satisfied, too.

If $x = y = \dfrac{1}{2}$ we get

$$F_{fx,fy}(\phi(t)) = \varepsilon_0(t - |0 - 0|) = \varepsilon_0\left(2t - \left|\dfrac{1}{2} - \dfrac{1}{2}\right|\right) = F_{x,y}(\phi(2t)),$$

thus the condition (2) is satisfied as well.

From the previous we conclude that the condition (2) of Theorem 3.1 is satisfied, thus the function $f(x)$ has a unique fixed point. It is easy to see that this point is $x = 0$.

Note that for function $f(x)$ Theorem 2.8 is not conclusive, for the case when $x \in \left[0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right]$ and $y = \frac{1}{2}$. We conclude that the result presented in Theorem 3.1 is an improvement of result presented in Theorem 2.8.

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