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ABSTRACTS OF TALKS PRESENTED AT THE INTERNATIONAL MATHEMATICAL CONFERENCE

TOPICS IN MATHEMATICAL ANALYSIS AND GRAPH THEORY (MAGT)

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1. INVITED TALKS

METHOD OF FACTORIZATION OF ORDINARY DIFFERENTIAL OPERATORS AND ITS APPLICATIONS

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The theme of lecture is connected to analytical and algebraic research of a problem of an integration of the ordinary differential equations. There are many papers devoted to this problem, they can be found in "Publications of the Faculty of Electrical Engineering, Series: Mathematics and Physics ETF".

It is necessary to tell, that mathematicians of Serbia and former Yugoslavia M. Petrović, D. Mitrinović, I. Bandić, B. Popov, P. Vasić (and others) have brought in the essential contribution to research of this inexhaustible problem. The urgency of a classical problem of integration ODE is connected not only to necessity of deriving of exact solutions for new mathematical models of natural-science problems, but also with necessity of testing of new numerical and analytical algorithms. The integrable equations (especially nonlinear) and methods of their solution to the beginning of XXI century have moved apart horizons in natural sciences.

The first part of the lecture is devoted to linear ODE the second and n-th orders. Has historically developed so, that two basic approaches to a problem of an integration

linear ODE methods of transformation of variables and factorizations of differential operators were.

The important role thus is played with transformations KUMMER-LIOUVILLE and EULER-IMSHENETSKY-DARBOUX, and also commutative differential operators. A differential algebra and the group analysis at which sources stood J. LIOUVILLE, S. LIE and D. MORDUKHAI-BOLTOVSKOI, have allowed to unit various approaches to a problem of an integrability. We shall remark also, that at research linear ODE the Significant value have the nonlinear equations such as RICCATI, ERMAKOV and KUMMER-SCHWARZ. In the lecture effectiveness of joint application of methods of a factorization of linear differential operators and transformations of variables is shown.

The second part of lecture is devoted nonlinear ODE. Factorizations of the nonlinear equations of second and higher order as through commutative, and noncommutative nonlinear differential operators are considered. The method of an exact linearization for the nonlinear equations n-th the order is explained also. Are indicated on some applications of the obtained results. The detailed account of a material of lecture can be found in books [**1, 2**].

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SPECIAL FUNCTIONS: THEIR APPROXIMATION AND BOUNDS

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The STEFFENSEN inequality and bounds for the ČEBYŠEV functional are utilised to obtain bounds for some classical special functions. The technique relies on determining bounds on integrals of products of functions. The above techniques are used to obtain novel and useful bounds for the BESSEL function of the first kind, the Beta function, and the Zeta function. Further, using convexity arguments, it is shown that Zeta at the odd integers is bounded by the Harmonic mean of Zeta at the immediate even integers.

INVERSE CONDUCTIVITY PROBLEMS IN THE ELECTRICAL NETWORKS

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In this talk, we discuss the inverse problem of identifying the connectivity and the conductivity of the links between adjacent pair of nodes in the electrical resistor network,

in terms of an input-output map. To do this we deal with an elliptic operator Δ_ω and an ω-harmonic function on the graph, with its physical interpretation as a diffusion equation on the graph, which models the electrical networks. After deriving the basic properties of ω-harmonic functions, we prove the solvability of (direct) problems such as the DIRICHLET and NEUMANN boundary value problems. Our main result is the global uniqueness of the inverse conductivity problem for a network under a suitable monotonicity condition.

In addition, as an application, we introduce the monotone method to identify a possible corrupted subregion in the electrical resistor networks..

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SOME PROPERTIES OF SIGNLESS LAPLACIAN EIGENVALUES OF GRAPHS

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Let G be a simple graph with n vertices. The characteristic polynomial $\det(xI-A)$ of a (0,1)-adjacency matrix A of G is called the *characteristic polynomial of* G and denoted by $P_G(x)$. The eigenvalues of A (i.e. the zeros of $\det(xI-A)$) and the spectrum of A (which consists of the n eigenvalues) are also called the *eigenvalues* and the *spectrum* of G, respectively.

Let n, m, R be the number of vertices, the number of edges and the vertex-edge incidence matrix of a graph G. The following relations are well-known: $RR^T = A + D$, $R^TR = A(L(G)) + 2I$, where D is the diagonal matrix of vertex degrees and $A(L(G))$ is the adjacency matrix of the line graph $L(G)$ of G. From these relations we immediately get $P_{L(G)}(\lambda) = (\lambda+2)^{m-n}Q_G(\lambda+2)$, where $Q_G(\lambda)$ is the characteristic polynomial of the

matrix $Q = A + D$. The polynomial $Q_G(\lambda)$ will be called the *Q-polynomial* of the graph G. The spectrum of Q will be called the *Q-spectrum*.

The matrix $L = D - A$ is known as the *Laplacian* of G and is very much studied in the literature. The matrix $A + D$ is called the *signless Laplacian* and appears very rarely in published papers.

The relation, derived above, provides a direct link between the spectra of line graphs and the Q-spectra of graphs. The computational evidence that Q-spectra perform better than the adjacency spectra supports the idea of using the spectrum of $L(G)$ instead of spectrum of G. In the other direction, the well developed theory of graphs with least eigenvalue -2 support the idea that, among matrices associated with a graph (generalized adjacency matrices), the signless Laplacian seems to be the most convenient to be used in studying graph properties.

We survey properties of spectra of signless Laplacians of graphs and discuss possibilities for developing a spectral theory of graphs based on this matrix. Some new results on the spectra of signless Laplacians are included as well. Among other things, we present eigenvalue bounds for several graph invariants, an interpretation of coefficients of the characteristic polynomial, a theorem on powers of the signless Laplacian and applications of the star complement technique.

NEW INEQUALITIES OF THE KANTOROVICH TYPE FOR BOUNDED LINEAR OPERATORS IN HILBERT SPACES

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Some new inequalities of the Kantorovich type are established. They hold for larger classes of operators and subsets of complex numbers than considered before in the literature and provide refinements of the classical results in the case when the involved operator satisfies the usual conditions. Several new reverse inequalities for the numerical radius of a bounded linear operator are obtained as well.

WHAT REALLY IS HADAMARD'S INEQUALITY?

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The Hadamard inequality usually stated as a result valid for convex functions only, actually holds for many other functions. We argue that an attempt ought to be made to close this gap by either changing the inequality or considering the measures in the integrals as a "second variable."

OUTER SOLAR SYSTEM ON THE EDGE OF CHAOS

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Over the recent years, many numerical integrations of the system of outer planets in our Solar System (Sun plus Jupiter, Saturn, Uranus, Neptune) have been performed. Unfortunately, there have been inconsistent results between investigators concerning the existence of chaos in this system. This raises the issue of the reliability of these integrations, since it is known that numerical integration with too-large a timestep can spuriously inject numerical chaos into a system which, in reality, is non-chaotic.

Luckily, it turns out that both sets of investigators are correct. In particular, we demonstrate that the system of Jovian planets, integrated for 200 million years as an isolated 5-body system using many sets of initial conditions all within the uncertainty bounds of their currently known positions, can display both chaos and near-integrability. The conclusion is consistent across four different integrators, including several comparisons against integrations utilizing quadruple precision.

Thus, given the current observational error in the positions of the outer planets, it is not possible at present to determine if the system is chaotic or not.

THE RANKIN-SELBERG PROBLEM

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The possibilities to improve the classical bound $\Delta(x) \ll x^{3/5}$ in the Rankin–Selberg problem by complex integration and Voronoï type formulas are discussed. The problem is the estimation of the error term function

$$\Delta(x) = \sum_{n\le x} c_n - Cx,$$

where the notation is as follows. Let $\varphi(z)$ be a holomorphic cusp form of weight κ with respect to the full modular group $SL(2,\mathbb{Z})$, denote by $a(n)$ the n-th Fourier coefficient of $\varphi(z)$, and let c_n be the convolution function defined by

$$c_n = n^{1-\kappa} \sum_{m^2|n} m^{2(\kappa-1)} \left| a\left(\frac{n}{m^2}\right)\right|^2,$$

while $C > 0$ is an explicit constant. A new mean square bound for $\Delta(x)$ is proved, namely

$$\int_1^X \Delta^2(x)\,\mathrm{d}x \ll_\varepsilon X^{1+2\beta+\varepsilon}, \quad \beta = \frac{2}{5-2\mu(1/2)},$$

where

$$\mu(\sigma) := \limsup_{t\to\infty} \frac{\log|\zeta(\sigma+it)|}{\log t}.$$

NONLOCAL SYMMETRIES: PAST, PRESENT AND FUTURE

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Nonlocal symmetries entered the literature in the Eighties of the last century largely through the work of Peter Olver. It was observed that there could be gain of symmetry in the reduction of order of an ordinary differential equation. Subsequently the reverse process was also observed. In each case the source of the 'new' symmetry was a nonlocal symmetry, *ie* a symmetry with one or more of the coefficient functions containing an integral. A considerable number of different examples and occurrences were reported by Abraham-Shrauner and Guo in the early Nineties. The role of nonlocal symmetries in the integration, indeed integrability, of differential equations was excellently illustrated by Abraham-Shrauner, Govinder and Leach with the equation

$$yy'' - y'^2 + f'(x)y^{p+2} + pf(x)y'y^{p+1} = 0$$

which had been touted as a trivially integrable equation devoid of any point symmetry. Further theoretical contributions were made by Govinder, Feix, Bouquet, Géronimi and others in the second half of the Nineties. This included their role in reduction of order using the nonnormal subgroup. The importance of nonlocal symmetries was enhanced by the work of Krause on the Complete Symmetry Group of the Kepler Problem. Krause's work was furthered by Nucci and there has been considerable development of the use of nonlocal symmetries by Nucci, Andriopoulos, Cotsakis and Leach. The determination of the Complete Symmetry Group for integrable systems such as the simplest version of the Ermakov equation, $y'' = y^{-3}$, which possesses the algebra $sl(2, R)$ has proven to be highly nontrivial and requires some nonintuitive nonlocal symmetries. The determination of the nonlocal symmetries required to specify completely the differential equations of nonintegrable and/or chaotic systems remains largely an open question.

FORMAL ORTHOGONAL POLYNOMIALS WITH RESPECT TO A MOMENT FUNCTIONAL AND APPLICATIONS

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Let a complex valued linear functional L be given on the linear space of all algebraic polynomials. We consider a sequence of polynomials $\{P_n(x)\}_{n=0}^{\infty}$ orthogonal with respect to L (formal orthogonal polynomial sequence) i give several applications. We study several quasi-definite cases when the moments of the functional L are complex numbers. Especially, we consider: (1) orthogonality on the semicircle and a circular arc; (2) orthogonality with respect to certain oscillatory weights; (3) orthogonality on the radial rays. A distribution of zeros, recurrence relations, asymptotics, etc. for such polynomials are presented, as well as some applications in numerical analysis, in particular in quadrature precesses.

USING GENE ONTOLOGY GRAPHS FOR BIOMARKER SELECTION FROM INTEGRATED MICROARRAY, PROTEOMICS AND CLINICAL DATA

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High throughput microarray and proteomics technologies can be extremely helpful in improving understanding of molecular mechanisms and biomarker selection by identification of genes and proteins that differentiate various groups of subjects. However, a high dimensional sample collected over a small number of subjects could significantly influence the results. Therefore, robust approaches for integration and analysis of data from high throughput technologies are necessary for successful knowledge extraction. In this talk we illustrate how uncertainty can be reduced by developing novel data analysis techniques that exploit prior domain knowledge early in the process. The proposed method for selection of biomarkers from microarray data is based on an assumption that a more accurate characterization is possible by analyzing deviations in expression of genes from a limited set of functions. The new method starts by selection of significantly differentially expressed genes followed by using Gene Ontology (GO) graphs to discover the ones that are highly overrepresented by the selection. Only the genes annotated with the most significant function are selected as biomarkers. In Chronic Fatigue Syndrome (CSF) identification experiments a Support Vector Machine that uses as attributes genes obtained by the proposed two-step process achieved higher accuracy than when using genes obtained by the traditional one-step selection. Benefits were also evident when using GO to effectively integrate microarray and proteomics data for CFS identification. Finally, we studied the clinical CFS data to discover factors that explain sources of CFS identification mistakes. We discovered significant difference in mental health, physical fatigue, and general fatigue indicators among cases differently classified by microarray and proteomics methods. This suggests that CSF identification could be improved by revising definitions of certain clinical conditions.

Reported results were obtained through a collaboration with Hongbo Xie and Slobodan Vucetic both from Information Science and Technology Center at Temple University.

INEQUALITIES: SOME PROBABILISTIC, SOME MATRIC, AND SOME BOTH

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It is interesting that a theory of "equations" has been developed in different areas, as for example, theory of differential equations, functional equations, linear equations. However, there are few general methods for obtaining a theory of "inequations" (inequalities). Some underlying themes are convexity, linear programming, Schur convexity and majorization. In this survey we exhibit a connection between some probabilistic inequalities and inclusion-exclusion results for eigenvalues. We also provide a new method for proving some probabilistic inequalities that carry over to matrix inequalities.

ALGEBRA OF GENERALIZED FUNCTIONS, GENERALIZED HYPERFUNCTIONS AND ALGEBRA OF MEGAFUNCTIONS

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First, the sheaf of algebras of generalized functions $\mathcal{G}$ is introduced and discussed. Then sheaves of spaces of generalized hyperfunctions $\mathcal{BG}$ and algebras of megafunctions $\mathcal{MG}$ are introduced. There exist injective sheaf homomorphisms $\mathcal{G} \to \mathcal{BG}$ and $\mathcal{BG} \to \mathcal{MG}$.

Examples through partial differential equations are given and discussed.

We refer to [**1**] for the general theory of algebras of generalized functions.

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MORSE MATCHINGS OF COLORED GRAPHS

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Recently R. Forman developed a discrete analog of the well-known Morse theory. M. Joswig and M. Pfegg have proven that computing optimal Morse matching is NP-complete. In this talk we explore the possibilities of generalizing the notion od Morse matching to several combinatorial structures, in particular, to Colored graphs. We present some preliminary results. [This is work in progress with Neža Mramor].

PROTEIN-PROTEIN INTERACTION NETWORKS: ISSUES, MODELS, AND COMPARISONS

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One of the fundamental problems in computational biology is understanding the inner workings of a cell. Most cellular processes are carried out by protein-protein interactions (PPIs). Thus, analyzing and modeling of large PPI networks is an integral part of this process. Analogous to biological sequence comparison, comparing cellular networks is an important problem that could provide insight into biological understanding and therapeutics. The full-scale comparison of two arbitrary networks is computationally intractable, because it contains the *subgraph isomorphism problem*, which is NP-complete. Thus, heuristic measures must be developed for network comparison.

We devise a highly constraining network comparison metric based on local structural properties that are a direct generalization of the degree distribution. We use this new metric to demonstrate that geometric random graphs better model PPI networks than do Erdös-Rényi, random scale-free, or Barabási-Albert scale-free networks. Our

new systematic measure of a network's local structure imposes a large number of similarity constraints on networks being compared. In particular, we generalize the degree distribution, which measures the number of nodes "touching" k edges, into graphlet degree distributions measuring the number of nodes "touching" k graphlets, where graphlets are small connected non-isomorphic induced subgraphs of a large network. Clearly, the degree distribution is the first one in the spectrum of graphlet degree distributions, since an edge is the only 2-node graphlet. We design a network "agreement" measure as a number in [0,1] that encompasses the 73 graphlet degree distributions of 2-, 3-, 4-, and 5-node graphlets. This measure is easily extendible to a greater number of constraints (i.e., graphlets) and the extensions are limited only by the available CPU.

ON THE MAZUR-ULAM THEOREM AND THE ALEKSANDROV PROBLEM FOR ISOMETRIC MAPPINGS

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In 1932, S. Mazur and S. Ulam proved the fundamental property that every isometry of a normed real vector space onto another normed real vector space is a linear mapping up to translation. In 1970, A. D. Aleksandrov posed the following problem: Under what conditions is a mapping of a metric space onto itself preserving unit distance an isometry? We will provide a generalization of the Mazur-Ulam Theorem and a number of results concerning the solution of the Aleksandrov problem. In addition several new open problems will be posed for further research in Mathematical Analysis and Geometry.

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STAR COMPLEMENTS AND EXTREMAL GRAPHS

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Let G be a graph of order n with an eigenvalue μ of multiplicity k. A *star complement* for μ in G is an induced subgraph of order $t = n - k$ without μ as an eigenvalue. Star complements exist for any eigenvalue of any graph [**2**]. If $t > 2$ and $\mu \neq -1$ or 0 then

$n \leq \frac{1}{2}t(t+1)$ (sharp for $t = 8$); and if further G is regular, then $n+1 \leq \frac{1}{2}t(t+1)$ (sharp for $t = 3, 7, 23$) [**1**]. The regular graphs for which the second bound is attained are precisely the extremal strongly regular graphs. Such graphs have independence number at most $4\mu^2 + 4\mu - 2$ when $\mu > 0$, and clique number at most $4\mu^2 + 4\mu - 2$ when $\mu < 0$. Star complements are used to show that here equality holds only in the known extremal strongly regular graphs.

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SOME FOX-WRIGHT GENERALIZED HYPERGEOMETRIC FUNCTIONS AND ASSOCIATED FAMILIES OF CONVOLUTION OPERATORS

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Here, in this lecture, we aim at presenting a systematic account of the basic properties and characteristics of several subclasses of analytic functions (with Montel's normalization), which are based upon some convolution operators on Hilbert space involving the Fox-Wright generalization of the *classical* hypergeometric ${}_qF_s$ function (with q numerator and s denominator parameters). The various results presented in this lecture include (for example) normed coefficient inequalities and estimates, distortion theorems, and the radii of convexity and starlikeness for each of the analytic function classes which are investigated here. We also briefly indicate the relevant connections of the some of the results considered here with those involving the Dziok-Srivastava operator.

Keywords: Analytic function classes, convex functions, starlike functions, Fox-Wright generalized hypergeometric function, classical normalization, Montel's normalization, Hadamard product (or convolution), Dziok-Srivastava operator, Hilbert space, bounded linear operators, extremal functions, radii of starlikeness and convexity.

COLORINGS OF HYPERGRAPHS WITH LOCAL CONDITIONS

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We survey results and open problems on vertex colorings of hypergraphs where conditions are given on the edges. Such models include, e.g., the following types.

- 'Mixed hypergraph': each edge is specified as a D-edge (that contains at least two distinct colors) or a C-edge (at least two—maybe all—of its vertices having the same color).
- Each edge E contains at least a given number $c(E)$ of colors.
- 'Hypergraph mixed coloring': each edge E contains at least $s(E)$ and at most $t(E)$ colors.
- The number of occurrences of each color in edge E is at least $a(E)$ and at most $b(E)$.
- 'Pattern hypergraph': for each edge, the family of allowed color partitions is given.

2. CONTRIBUTED TALKS

ON MICCHELLI COMBINATION OF MODIFIED BERNSTEIN POLYNOMIALS

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The modified Bernstein polynomial operators

$$M_n(f;t) = (n+1)\sum_{k=0}^{n} p_{n,k}(t)\int_0^1 p_{n,k}(u)f(u)\,\mathrm{d}u,$$

where $p_{n,k}(t) = {}^nC_k t^k(1-t)^{n-k}$, $0 \le t \le 1$, defined on $L_B[0,1]$, the space of bounded and integrable functions on $[0,1]$ were introduced by Durrmeyer and extensively studied by Derriennic [**1**] and several other researchers. It turns out that the order of approximation by these operators is, at best, $O(n^{-1})$ however smooth the function may be. In order to improve this rate of convergence we apply the technique of iterative combination given by Micchelli [**2**]. The object of this paper is to study direct theorems in ordinary as well as in simultaneous approximation for these combination of the operators M_n.

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NORMAL CAYLEY DIGRAPHS WITH VALENCY 2 ON GROUPS OF ORDER *pq*

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For a group G, and a subset S of G such that $1_G \notin S$, let $\Gamma = Cay(G, S)$ be a Cayley digraph. We call a Cayley digraph $\Gamma = Cay(G, S)$ normal for G if G_R, the right regular representation of G, is a normal subgroup of the full automorphism group $Aut(\Gamma)$ of Γ. In this paper we determine the normality of Cayley digraphs of valency 2 on the groups of order pq.

Keywords : Cayley digraph, normal Cayley digraph, arc-transitive Cayley digraph.

ON THE MATRIX EQUATION $XA - AX = \tau(X)$

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We study the matrix equation $XA - AX = \tau(X)$ in $M_n(K)$, where τ is an automorphism of field K of finite inner order k. It is shown that this equation has a nonzero solution if and only if the matrix $N_k(E \times \tau^{-1}(A) - \tau^{-1}(A) \times E)$ has eigenvalue $\lambda = 1$. In case when $k = 1$ this criterion boils down to already known result.

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NOTE ON ASYMPTOTIC CONTRACTIONS

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W. A. Kirk first introduced the notion of asymptotic contractions and proved the fixed point theorem for this class of mappings. In this talk (paper) we present a new short and simple proof of Chen's theorem which generalize Kirk's result.

A FAST ALGORITHM FOR LINEAR CONVOLUTION OF DISCRETE TIME SIGNALS

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A new, computationally efficient, algorithm for linear convolution is proposed. This algorithm uses an N point instead of the usual point circular convolution to produce a linear convolution of two N point discrete time sequences. To achieve this, a scaling factor is introduced which enables the extraction of the term representing linear convolution from any algorithm that computes circular convolution. The proposed algorithm is perfectly accurate provided that the chosen circular convolution algorithm does not introduce round off errors. The analysis is supported by simulation examples for several typical application cases.

ON THE NUMBER OF CERTAIN TYPES OF RESTRICTED PERMUTATIONS

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We show a technique for generating a system of linear recurrence equations that enumerate the number of restricted permutations satisfying the conditions $-k \leq p(i)-i \leq r$ (for arbitrary natural numbers k and r) and $p(i) - i \notin I$ (for an arbitrary set I). We solve several types of restricted permutations and in one case we find the Fibonacci $(r + 1)$-step number. We establish the bijection between the number of compositions and the number of restricted permutations. We also show that computing the number of restricted permutations using our techniques is computationally much more efficient than expanding the permanent. Using a program that implements our technique, we have contributed over sixty sequences to the Sloane's online encyclopedia of integer sequences.

ORDERING GRAPHS WITH THE INDEX IN THE INTERVAL $\left(\mathbf{2}, \sqrt{\mathbf{2}+\sqrt{\mathbf{5}}}\right)$

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We consider the problem of ordering graphs by the index (the largest eigenvalue or spectral radius of the adjacency matrix) in the class of connected graphs with a fixed order n and index being in the interval $\left(2, \sqrt{2+\sqrt{5}}\right)$. For any fixed n (if not too small), we give the ordering for many graphs under considerations provided their index is located near to the end points of the above interval.

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GRAPHS FOR WHICH THE LEAST EIGENVALUE IS MINIMAL

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Many results have been proved concerning graphs whose largest eigenvalue is maximal, or minimal. In contrast, there are far fewer results for the least eigenvalue. Let G be a connected graph with n vertices and e edges whose least eigenvalue is minimal, and let $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ be a corresponding eigenvector. Let V^-, V^0, V^+ be the subsets of the vertex set of G consisting of those vertices u for which $x_u < 0$, $x_u = 0$, $x_u > 0$, respectively. We show that consideration of these sets is helpful in deriving some structural results.

GRACEFUL LABELLING OF TREES WITH MAXIMUM DEGREE 3

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A graceful labelling of a finite undirected graph G with n edges is a one-to-one function from the set of vertices of G to the set $\{0, 1, 2, \ldots, n\}$ such that the induced edge labels are all distinct. This labelling was originally introduced in 1967 by Rosa who also showed that the existence of a graceful labelling of a given graph G with n edges is a sufficient condition for the existence of a cyclic decomposition of a complete graph of order $2n + 1$ into subgraphs isomorphic to the given graph G. The famous Graceful Tree Conjecture (also known as Ringel-Kotzig, Rosa's, or even Ringel-Kotzig-Rosa conjecture) says that all trees have a graceful labelling. In this talk we explore bipartite graceful labellings for trees with maximum degree 3.

A UNIFIED EXPLICIT FORMULA FOR STIRLING NUMBERS

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In this paper we consider the explicit expression for the new arrays of numbers. These numbers are generalized Stirling numbers. Various results related to those special numbers due to Carlitz, Howard and Koutras, and others, respectively, are consequence of our results. Related recurrence relations and related polynomials will be also considered.

POTENTIAL THEORY FOR BVP'S IN FINITE NETWORKS

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We aim here at analyzing Self-adjoint Boundary Value Problems on finite networks associated with positive-definite Schrödinger operator. In this discrete setting, such

operator can be interpreted as an integral operator and therefore a discrete Potential Theory with respect to its associated kernel can be built. We prove that the SCHRÖDINGER kernel satisfies enough principles to assure the existence of equilibrium measures for any proper subset. These measures are used to obtain systematic expressions of the resolvent kernels associated with the different BVP's.

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OPTIMAL VENTCEL GRAPHS, MINIMAL COST SPANNING TREES AND ASYMPTOTIC PROBABILITIES

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Let X_n^ϵ be an irreducible MARKOV chain with a finite state space S and transition function $p^\epsilon(i,j) = p_{i,j}\epsilon^{u(i,j)}$ where $0 \leq U(i,j) \leq \infty$ is a cost function. (We assume $p_{i,j} = 0$ if and only if $u(i,j) = \infty$.) It has been shown [**1**] that independent of the initial distribution, there are constants $h(i)$ and β_i and $\lim\limits_{\epsilon\to 0}\lim\limits_{t\to\infty} \dfrac{P(X_t^\epsilon = i)}{\epsilon^{h(i)}} = \beta_i$.

Let $\underline{S}$ be the set of all states in S with $h(i) = 0$, i.e., $\underline{S} = \{i \in S, h(i) = 0\}$. $\underline{S}$ is called the global minimum set. Various asymptotic probabilities related to $\underline{S}$ have been established in [**2**]. Among others, starting with the invariant distribution, the expected hitting time $E^\epsilon T$ of $\underline{S}$ is of order $\epsilon^{-\delta}$ and the invariant distribution $\mu^\epsilon(i)$ is of order $\epsilon^{d(i)}$ where the constants δ and $d(i)$ are expressed in terms of a complicated hierarchy of "cycles" related to the cost function $U(i,j)$. In this paper, we shall express these constants in terms of VENTCEL graphs (minimum cost spanning trees) to simplify the concept and computation of these constants. We also establish some new properties of VENTCEL graphs.

KEYWORDS: Ventcel graphs, cycles, asymptotic probability, minimal cost spanning tree.

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BOUNDARY VALUE PROBLEMS FOR DIFFUSION AND ELASTIC OPERATORS ON NETWORKS

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In this talk, we discuss discrete versions of the heat equations and the wave equations, which are called the ω-diffusion equations and the ω-elastic equations on graphs. After deriving some basic properties, we solve the ω-diffusion equations under:

1. the condition that there is no boundary, 2. the initial condition and 3. the Dirichlet boundary condition.

We also give some additional interesting properties on the ω-diffusion equations, such as the minimum and maximum principles, Huygens property. Analogues of the ω-elastic equations on graphs are also discussed.

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STURM'S THEOREMS THROUGH ITERATIONS

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The classical Sturm's results apply to linear homogeneous differential equation of the second order in the case of existence of oscillatory solutions. Sturm's basic theorem states that the equation has two linearly independent oscillatory solutions and that zeroes of one solution are interlaced with extremes of the other solution and vice versa. Together with continuity, this gives the basic notion of oscillations. The Sturm's theorem gives no further details on the locations of zeroes, as well as on the values of extremes.

We present the value of the iterations method. It is actually the classical iterations method, with coefficients which are not determined but very general, and have least one general feature, beside continuity. For example, the coefficient $a(x)$ is always positive and is not too small. The location of zeroes can then be well determined, implying the same for extremes. It also provides for research of asymptotics and, finally, stability of the solutions, the latter being of basic interest for engineering.

We stress that are just a few results on locations of zeroes and extremes in literature, and no results in bigger courses and handbooks. This fact surprises, due to their importance with oscillations.

THE CONCEPT OF TUNER SET FOR GRAPHS

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The idea of equilibrium of a graph G was conceived to observe how the vertex degree distribution affects the average degree of the graph, $d(G)$, and it was initially applied to maximal outerplanar graphs (mops). In this work, the idea is formally defined and it is extended to graphs in general. We call tuner set a subset of vertices of G with degree lower than $d(G)$ that are able to compensate the presence of vertices with degree greater than $d(G)$. We show that a graph can have distinct tuner sets. Our main result is the characterization of tuner sets for graphs that have the average degree as an integer number. For these graphs, the following properties hold: *The tuner set is unique and it is composed by all vertices with degree lower than* $d(G)$. This subject turns out particularly interesting when dealing with Hamiltonian graphs. Let H be a Hamiltonian graph with n vertices. If H is a simple cycle, that is, the Hamiltonian graph with the smaller number of edges, H is a 2-regular graph and its tuner set is empty. For each n, when we add edges to H (provided that $H \neq K_n$), we have at least one instance which meets the properties above.

I AND I* - CONVERGENCE IN TOPOLOGICAL SPACES

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Recently Kostyrko et. al. introduced the concepts of I and I* - convergence of real sequences as a generalization of statistical convergence of sequences (introduced by Fast and then Schoenberg) which is based on the structure of the ideal I of subsets

of the set of natural numbers. In this paper these concepts are extended to an arbitrary topological space and several basic properties of these concepts are studied. Also the relationship between the two types of convergences are established where it is shown that in general I* convergence implies I convergence but the converse is not true. However if the ideal satisfies some additional condition (condition AP) then these two concepts become equivalent.

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In memory of Professor Mitrinović:

ON THE NEED FOR, AND IMPORTANCE OF A THEMATIC MONOGRAPH EXCLUSIVELY ON PERIODICAL SOLUTIONS OF DIFFERENTIAL EQUATIONS

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Our dear Professor Mitrinović had been pleading for many years for an idea and looking for associates for a supplemented Kamke-like book on differential equations, due to their huge importance and accelerated development during the second half of the 20^{th} century.

Due to boosted development of the engineering, along with mathematics and especially differential equations, huge hops occurred leaving little time for some methodology, stabilization of textbooks, and extraction of the most essential results out of broad literature.

For example, there is the very developed theory of stability of non-linear equations and systems in Electronics nowadays (Van der Paul, Lienard, Rayleigh) as seen in the contemporary magazines; yet there are no simple fundamental theorems, at least on sufficient conditions for a differential equation to have a periodical solution. The huge void between higher trigonometry courses (for example the famous Novoselov: The Special Course in Trigonometry) and contemporary research is therefore noticeable, hardening both theoretical and practical work in those fields.

In the approximately 500 pages long manuscript there are approximately 200 theorems given. Those are of a fundamental character, and not of a special profoundness, but still of quite a generality. Some of the theorems are presented in brief.

GLOBAL ASPECT OF NON-HOMOGENOUS DIFFERENTIAL EQUATION OF THE SECOND ORDER

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Without specifying the coefficients $a(x)$ and $b(x)$ of the internal system of the differential equation, as well as the external influence $f(x)$, we try to give a general analysis of solutions under very general conditions, in order to overcome the delicate classical way of solving differential equations by quadratures. Continuity and Lipschitz's condition as general conditions apply at the moment, as well as some class-wise conditions: sign, or limitedness only, or a determinant, a discriminant, and similar.

Using the iterations method we obtain surprisingly good and general solutions. For example, for the equation $y''+a(x)y'+b(x)y=f(x)$ the solution is a sum of effects in the form: $y(x)=I_a+I_b+I_{a,b}+I_f+I_{a,f}+I_{b,f}$, where I_z are corresponding iterations. However, other approaches are possible, employing other fundamental functions, for example the functions

$$P=b(x)\exp\left(\int a(x)\,\mathrm{d}x\right),\qquad Q(x)=\exp\left(-\int a(x)\,\mathrm{d}x\right).$$

In this case the solution has the form $y(x)=I_P+I_Q+I_{P,Q}$.

Since the functions P and Q are very characteristic for linear differential equations of the n-th order, we believe those procedures are very promising.

ON A LITTLEWOOD-PALEY TYPE INEQUALITY

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It is proved the following: If u is function harmonic in the unit ball $B\subset\mathbb{R}^N$, and $0<p<1$, then there holds the inequality

$$\sup_{0<r<1}\int\limits_{\partial B}|u(ry)|^p\,\mathrm{d}\sigma\le|u(0)|^p+C_{p,N}\int\limits_B\left(1-|x|\right)^{p-1}\left|\Delta u(x)\right|^p\,\mathrm{d}V(x)$$

This proved in the case $p>(N-2)/(N-1)$.

ON ONE SET OF INEQUALITIES

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Suppose that m and n are positive integers, with $n<m$, and $V=\{x_1,\ldots,x_m\}$ is the set of variables. We consider the set of inequalities of the form $x_{i_1}+\cdots+x_{i_{n_i}}\ge 1-1/n$,

where $x_{i_1}, \ldots, x_{i_{n_i}}$ are distinct variables from V, such that every n inequalities have a common variable. It is shown whether every system of the above form has a solution which satisfies conditions: $x_1 \geq 0, \ldots, x_m \geq 0$, $x_0 + \cdots + x_m = 1$. This question solves a problem in probabilistic logic.

LINEARIZABILITY CONDITIONS OF A POLYNOMIAL SYSTEM OF DEGREE FIVE

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We study the problem of linearizability for plane polynomial systems of ordinary differential equations. Our main result is the following theorem which gives the necessary and sufficient conditions for linearizability of the ten-parameter system of degree five

$$\begin{aligned} \dot{x} &= x(1 - a_{20}x^2 - a_{11}xy - a_{02}y^2 - a_{-13}x^{-1}y^3 - a_{22}x^2y^2), \\ \dot{y} &= -y(1 - b_{20}x^2 - b_{11}xy - b_{02}y^2 - b_{3-1}x^3y^{-1} - b_{22}x^2y^2). \end{aligned} \tag{1}$$

Theorem 1. *The system* (1) *is linearizable if and only if one of the following conditions holds*:

1) $b_{11} = b_{20} = b_{22} = a_{22} = a_{-13} = 3a_{02} + b_{02} = a_{11} = a_{20} = 0$,

2) $b_{11} = b_{20} = b_{22} = a_{22} = b_{3,-1} = a_{11} = a_{20} = 0$,

3) $b_{11} = b_{22} = a_{22} = 7a_{02} + 3b_{02} = a_{11} = 3a_{20} + 7b_{20}$
$= 21a_{-13}b_{3,-1} + 16b_{20}b_{02} = 112b_{20}^3 + 27b_{3,-1}^2 b_{02} = 49a_{-13}b_{20}^2 - 9b_{3,-1}b_{02}^2$
$= 343a_{-13}^2 b_{20} + 48b_{02}^3 = 0$,

4) $b_{02} = b_{11} = b_{22} = a_{22} = b_{3,-1} = a_{02} = a_{11} = a_{20} + 3b_{20} = 0$,

5) $b_{02} = b_{11} = b_{22} = a_{22} = a_{-13} = a_{02} = a_{11} = 0$,

6) $b_{11} = b_{22} = a_{22} = b_{3,-1} = a_{-13} = a_{02} + b_{02} = a_{11} = a_{20} + b_{20} = 0$,

7) $b_{11} = a_{22} = b_{3,-1} = a_{-13} = a_{02} = a_{11} = b_{20}b_{02} + b_{22} = 0$,

8) $b_{11} = b_{20} = b_{22} = b_{3,-1} = a_{-13} = a_{11} = a_{20}a_{02} + a_{22} = 0$.

ON PSEUDOSCALAR PRODUCT OF ANISOTROPIC VECTORS

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It is shown in this paper that pseudoscalar product of two unisotropic vectors can be represented as

$$a \wedge b = |a|\,|b| \cos \Psi, \qquad \Psi \in \mathbb{C}.$$

The results of the work is presented by two theorems and geometrically interpreted.

POLYNOMIAL INTERPOLATION PROBLEM FOR SKEW POLYNOMIALS

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Let $R = K[x;\sigma]$ be the skew polynomial ring over division ring K. We will introduce the value of derivatives of skew polynomial at scalar. Analogue definition of derivatives of commutative polynomials $K[x]$ as function of $K[x] \to K[x]$ is not possible in non-commutative case. That is the reason why we define the value of derivative of skew polynomial at scalar. It is based on the properties of skew polynomial ring, and it implies some useful theorems about them. The main result of this paper is generalization of polynomial interpolation problem for skew polynomials. We will find conditions under which one can find unique polynomial of degree less then n which takes some values at $x_i \in K$ $(1 \leq n)$. We will use K-linear function of left K-space $K[x;\sigma]$. We can find and some kind of Silvester-Lagrange skew polynomial.

LOCATION OF EIGENVALUES OF ACYCLIC MATRICES

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Motivated by a discrete version of Christoffel-Darboux identity, in this talk we present some results on the location of the eigenvalues of acyclic matrices.

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ON FIXED POINT IN D-METRIC SPACES

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In 1992 B. C. Dhage introduced a new structure of so called D-metric spaces and proved some fixed point results for contractive mappings. In this lecture we are going to talk about fixed point results for mappings with generalized contractive condition on D-metrics spaces.

OUTER SOLAR SYSTEM ON THE EDGE OF CHAOS THE PROPERTY OF GOOD DECOMPOSITION FOR SLOWLY VARYING FUNCTIONS

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It is well known that for slowly varying functions the sum, product and quotient of two slowly varying functions are always slowly varying, but their difference need not be slowly varying. We study the following problem: given a nondecreasing slowly varying function L, which is the sum of two nondecreasing functions $L = F+G$, find the conditions that guarantee that the components F and G are necessarily slowly varying. If this holds for a class of slowly varying functions, we say that this class has the property of good decomposition. We present conditions that characterize the property of good decomposition for some classes of slowly varying functions.

COMMON FIXED POINT THEOREMS FOR R-WEAKLY COMMUTING MAPPINGS DEFINED ON FUZZY METRIC SPACES

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This paper is to present some fixed point theorems for R-weakly commuting mappings defined on complete Fuzzy metric spaces, under different concepts of completness, in both the sense of M. Grabiec [**3**], and George and Veeramani [**1**].

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TRANSVERSAL SPACES AND FIXED POINT THEOREMS

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In this paper we shall define Transversal spaces (upper and lower) as a natural extension of Metric spaces, Probabilistic metric spaces and Fuzzy metric spaces. Also, we formulate and prove some fixed and common fixed point theorems, under different conditions, and discuss various generalizations of commutativity and their applications to Fixed point theory.

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ON A DIFFERENTIAL EQUATION WITH NONSTANDARD COEFFICIENTS

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In this paper some classes of nonlinear differential equation with coefficients generalized functions are considered. The classical distribution theory turn out to be insufficient for treating problems which accommodates differential equations, nonlinear operations and singularities. That's why we are using the framework of Colombeau algebras of generalized functions. The basic idea underlying Colombeau's theory in its simplest form is that of embedding the space of distributions into a factor algebra of $\mathcal{C}^\infty(\mathbf{R}^n)^I$, $I = (0,1)$ with regularization by convolution with a fixed "mollifier" ϕ. Using this theory the existence and uniqueness of the solution of the differential equations are given.

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CONVEX FUNCTIONS - SOME HISTORICAL NOTES

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In the communication some new material for the history of convex functions with special reference to history of the second order criteria will be given.

(p,w)-HARMONIC FUNCTIONS AND INVERSE PROBLEMS ON NONLINEAR NETWORKS

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The purpose of this talk is to show the inverse problem which is to identify the conductivity of links between adjacent pairs of nodes in a nonlinear network. To do this, we deal with the p-Laplacian operator $\Delta_{p,w}$ and (p,w)-harmonic functions on nonlinear networks. After deriving the basic properties of (p,w)-harmonic functions, we prove the solvability of (direct) problems such as the Dirichlet and Neumann boundary value problems. Using these properties, we show the global uniqueness of the inverse conductivity problem for a nonlinear network under a suitable monotonicity condition.

IMPLEMENTATION TABLEAU ALGORITHM FOR A DESCRIPTION LOGIC USING A MODEL TRANSFORMATION

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The paper presents implementation details of a tableau algorithm for concept satisfiability in ALCN description logic. The implementation relies on OMG's description logics metamodel, tableau metamodel and model transformation using Atlas Transformation Language (ATL) as a language for model transformations. The idea can be easily applied to implementation of a reasoning algorithm based on tableau for other non-classical logics.

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STABILITY FOR CUBIC FUNCTIONAL EQUATION IN THE SPACES OF GENERALIZED FUNCTIONS

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Stability problem of a cubic functional equation

$$f(x+2y)+3f(x)=3f(x+y)+f(x-y)+6f(y) \tag{1}$$

was proved by Rassias [**2**]. Jun and Kim [**1**] introduced different type of a cubic functional equation

$$f(2x+y)+f(2x-y)=2f(x+y)+2f(x-y)+12f(x) \tag{2}$$

which is equivalent to the equation (1). They established the general solution and also investigated Hyers-Ulam-Rassias stability for equation (2).

In this talk, we reformulate and prove the stability theorem of the equation (2) in the spaces of some generalized functions such as the space $\mathcal{S}'$ of tempered distributions and the space $\mathcal{F}'$ of Fourier hyperfunctions.

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SOME RESULTS ON CONJUGATE INTEGRAL GRAPHS

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Let G be a simple graph of order n. Let $c=a+\sqrt{bpm}$ and $\overline{c}=a-\sqrt{bpm}$, where a and b are two nonzero integers and m is a positive integer such that m is not a perfect square. We say that $A^c=[c_{ij}]$ is the conjugate adjacency matrix of the graph G if $c_{ij}=c$ for any two adjacent vertices i and j, $c_{ij}=\bar{c}$ for any two nonadjacent vertices i and j, and $c_{ij}=0$ if $i=j$. The conjugate spectrum of G consists of the eigenvalues $\lambda_1^c \geq \lambda_2^c \geq \cdots \geq \lambda_n^c$ of its conjugate adjacency matrix A^c. Further, we say that G is a conjugate integral graph if its conjugate eigenvalues are in the form $\lambda_i^c=p_i+q_i\sqrt{m}$ for $i=1,2,\ldots,n$, where p_i

and q_i are integral values. Besides, we say that G is a strongly conjugate integral graph if $q_i = 0$ for $i = 1, 2, \ldots, n$. We prove that if G is strongly conjugate integral then G is a regular graph which is cospectral to its complement $\overline{G}$. We also demonstrate that any non-integral strongly regular graph is strongly conjugate integral.

PARTIAL UNIMODALITY FOR INDEPENDENCE POLYNOMIALS OF SOME COMPOUND GRAPHS

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If s_k denotes the number of stable sets of size k in G, then the polynomial $I(G;x)$ with s_k as coefficients is the independence polynomial of G [**1**].

We show that $s_{\lceil(3\alpha-2)/5\rceil} \geq \cdots \geq s_{\alpha-1} \geq s_\alpha$ and $s_0 \leq s_1 \leq \cdots \leq s_{\lfloor(2\alpha+2)/5\rfloor}$ are valid for any graph H obtained from a graph G by appending two pendant edges to each vertex of G, where α is the size of a maximum stable set in H.

In [**2**] it has been conjectured that $I(T,x)$ is unimodal for any tree T. In particular, when trees are under investigation, the partial unimodality phenomenon we revealed may be considered as a step in an attempt to validate the conjecture.

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AN ALGEBRAIC APPROACH TO REDUNDANCY ANALYSIS OF SEQUENT PROOFS

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Proof search often involves a choice between alternatives which may result in redundant information once the search is complete. This behavior can manifest itself in proof search for sequent systems by the presence of redundant formulae or subformulae in a sequent for which a proof has been found. In this paper we investigate the detection

and elimination of redundant parts of a provable sequent by using labels and Boolean constraints to keep track of usage information. We illustrate our ideas in propositional linear logic, but we believe the general approach is applicable to a variety of sequent systems, including other resource-sensitive logics. Our work is intended as a contribution to a library of automatic support tools for managing redundancies in sequent calculi proof search. Our work is more aimed at producing proof assistants, rather than proof checkers or proof generators (ie. theorem provers).

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A METHOD OF PROVING A CLASS OF INEQUALITIES VIA PADE APPROXIMATIONS

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We consider a method of proving a class of analytical inequalities via Pade approximations. Our algorithm is based on Pade approximations and Remez algorithm for obtaining some boundary functions of appropriate functions which figure in the considered analytical inequalities. We obtain some new and well-known inequalities.

BAYESIAN COMMUNICATION LEADING TO NASH EQUILIBRIUM

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In decision theoretical situations among a group of players, the concept of mixed strategy Nash equilibrium has become central. Yet little is known the process by which players learn if they do. We shall give a protocol run by the mutual learning of their beliefs of players' actions, and focus the Bayesian aspect on a communication between the players with missing some information. We present the Bayesian belief revision process according to a communication graph, which leads to a mixed strategy Nash equilibrium for a strategic form game. We highlight the topological structure of the communication-

graph. It is shown that the acyclic condition of the communication-graph plays no role for inducing a NASH equilibrium.

Let us consider the following protocol: The players start with the same prior distribution on a state-space. In addition they have private information given by a partition of the state space. Beliefs of players are posterior probabilities: A player p-believes (simply, believes) an event with $0 < p \leq 1$ if the posterior probability of the event given his/her information is at least p. Each player predicts the other players' actions as his/her belief of the actions. He/she communicates privately their beliefs about the other players' actions through messages, and the receivers update their belief according to the messages. The players are assumed to be rational and maximizing their expected utility according their beliefs at every stage. Each player communicates privately his/her belief about the others' actions as messages according to a protocol, and the receivers update their private information and revise their belief. We show that

Main theorem. *Suppose that the players in a strategic form game have the p-belief system with a common prior distribution. In a communication process of the game according to a protocol with revisions of their beliefs about the other players' actions, the profile of their future predictions induces a mixed strategy* NASH *equilibrium of the game in the long run, even if the communication-graph is not acyclic.*

A FREE BOUNDARY PROBLEM BETWEEN TWO PARALLEL PLANES

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We study surfaces that are solutions for a geometrical model for thin films with gravity, are located between two parallel horizontal planes, and have a prescribed volume. We work with the energy functional that contains area and a gravitational energy and it is known that a solution to this free boundary problem has mean curvature which is a linear function of its height above a horizontal plane. We obtain some information about these surfaces and the question of their stability or instability.

SOME PROPERTIES OF POSYNOMIAL RINGS

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The notion of a posynomial (positive polynomial) appeared in geometric programming as a generalization of a polynomial. Zener introduced posynomial functions about forty years ago in order to compute minimal costs. Aside from economy and management, in the last decade posynomials have been used in optimal integral circuit design.

The applicability of posynomials essentially relies on definability of root functions in the theory of real closed fields (RCF) and on realtime procedures for quantifier elimination in RCF based on the partial cylindrical algebraic decomposition.

We shall study here some algebraic and computational properties of rings of posynomials over a commutative domain. In particular, it is proved that a posynomial ring $\mathrm{Pos}(\mathbf{R}, \mathbf{G})[\bar{x}]$ is not noetherian and it is not UFD (unique factorization domain) if $\mathbf{R}$ is a domain and $\mathbf{G}$ is an abelian group such that $\bigcup_{n>1} G_n \neq \{0\}$, where $G_n = \bigcap_{k\in\mathbb{N}} n^k G$. Further, we introduce the posynomial Zariski topology and prove the analogues to the Hilbert's Nullstellensatz and the real Nullstellensatz. Finally, we shall study the ideal membership problem in the posynomial rings $\mathrm{Pos}(\mathbf{K}, \mathbb{Z})[\bar{x}]$ and $\mathrm{Pos}(\mathbf{K}, \mathbb{Q})[\bar{x}]$ under assumption that $\mathbf{K}$ is a computable domain.

STOCHASTIC MODELLING OF THE GROWTH PROCESS

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This paper solves the problem of forecasting the number of trees in the selection stands predicted for harvesting in a future period, so that the present resource of the number of trees is sustained. This is achieved by stochastic modelling of the number of trees and the number of felled trees and by solving the partial differential equation. The same problem is solved in the papers [**4**] and [**5**]. Here the other mathematical model is represented.

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FIXED CHARGED NETWORK FLOW PROBLEM AND ITS COOPERATIVE GAME

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We introduce some variations of Fixed Charged Network Flow Problem. These problems for a complete graph are $\mathcal{NP}$-hard problems in terms of computing complexity theory. And we show that we can solve them for special graphs in polynomial time. One of the graphs is a kind of "series-parallel graph". If we have time, I will show a discrete convexity for the special case of this problem.

Also, we define a co-operative game for the problem. We can apply this game to Internet optical fiber network design or public electric networks. Generally, it is hard to compute Shapley value, nucleolus and so on. We define a new solution of the game for easy computation and show its characteristics.

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POISSON'S SUMMATION FORMULA FOR BOEHMIANS

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One form of Poisson's summation formula for a well-behaved function is given by

$$2\pi \sum_{n=-\infty}^{+\infty} f(x+2\pi n) = \sum_{n=-\infty}^{+\infty} \widehat{f}(n)e^{inx},$$

where

$$\widehat{f}(x) = \int_{-\infty}^{\infty} f(t)e^{-ixt}\mathrm{d}t$$

.

The importance of the Poisson summation formula is well-known. It has been found to be useful in many areas of mathematics, such as number theory, differential equations, and signal analysis. We will discuss the Poisson summation formula for a space of generalized functions $\beta_\ell(\mathbb{R})$. The space $\beta_\ell(\mathbb{R})$ contains a subspace which can be identified with the space of Lebesque integrable functions on the real line $\mathbb{R}$. It also contains all distributions having bounded supports. Furthermore $\beta_\ell(\mathbb{R})$ does contain some elements which are not distributions.

THE SIZE OF SOME ANTICHAINS FOR MULTISETS

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This paper is dealing with the rank numbers in the poset of the subsets of a multiset. For the largest of them I. Anderson gave an asymptotic formula. However, we have obtained explicit closed formulas for all ones. These formulas represent one generalization of Sperner's theorem. Also, many of known combinatorics results, for example an consekvence of Clements-Griggs Theorem, can be obtained by using them.

THE CHIP-FIRING GAME AND THE DIRICHLET GAME ON WEIGHTED GRAPHS

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The rule of a chip-firing game on a graph is as following: Starting from a given chip configuration, if a vertex v has at least as many chips as its degree, we can fire v by sending one chip along each edge from v to its neighbors. The game continues until no vertex can be fired.

The chip-firing game has been studied previously in terms of classification of legal game sequences, critical configurations, chromatic polynomials, and the Tutte polynomial.

It has been studied on a graph by several authors([**1**], [**2**]) that the final configuration of the game does not depend on the order of the firing but on the initial configuration and they were given an upper bound of the number of firing.

In this talk, we will prove the above result on a graph where nonnegative weight is given on each edge by generalizing the rule of the game and give a better bound than those studied before, we will also study the relation between the generalized rule and the original rule of the chip-firing game. Finally, we will deal with the chip-firing game with Dirichlet boundary condition.

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HERMITE EXPANSION OF ULTRADISTRIBUTION

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By characterizing Hermite expansions of elements of the generalized Gelfand-Shilov space, in quasianalytic and non-quasianalytic case, we give an unifying approach to the investigation of the Gelfand-Shilov spaces, extended Fourier hyperfunction space and spaces of tempered ultradistributions of Roumieu-Komatsu type. Similarly, we obtain an unifying approach to the Pilipović space, the space of Fourier hyperfunctions, and the space of tempered ultradistributions of Beurling-Komatsu type. As an application of the characterizations we give a simple proof of the kernel theorem, which imply that the representation of the Heisenberg group and the Weyl transform can be extended on generalized Gelfand-Shilov spaces.

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ON THE NEWTON-LIKE METHOD FOR THE INCLUSION OF POLYNOMIAL ZEROS

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The aim of this paper is to present an iterative method of Newton's type for the inclusion of an isolated complex zero of a polynomial. A special attention will be paid to the construction of computationally verifiable initial condition that provides the guaranteed convergence of the proposed method.

Let be given a disk $A := \{z : |z-a| \le \eta\}$ that contains only one simple zero ζ of an n-th degree polynomial P. All other zeros are supposed to lie in the exterior of the disk $A = \{a;\eta\}$. Starting from the initial disk $Z^{(0)} = \{a;\eta\} = A$, we construct the following iterative method for the inclusion of the wanted zero of the given polynomial P :

$$Z^{(m+1)} = z^{(m)} - \frac{1}{\dfrac{P'(z^{(m)})}{P(z^{(m)})} - (n-1)\Big\{h^{(m)}; d^{(m)}\Big\}} \quad (m = 0, 1, \ldots), \tag{1}$$

where

$$h^{(m)} = \frac{\bar{a} - \bar{z}^{(m)}}{\eta^2 - |z^{(m)} - a|^2}, \quad d^{(m)} = \frac{\eta}{\eta^2 - |z^{(m)} - a|^2}, \quad z^{(m)} = \operatorname{mid} Z^{(m)}.$$

The guaranteed convergence, the enclosure of zero and the quadratic convergence are stated in:

Theorem. *Let the sequence of disks* $\{Z^{(m)}\}$ *be defined by the iterative method* (1)*, assuming that the initial disk* $Z^{(0)} = \{a;\eta\}$ *is chosen so that the condition*

$$\Big|\frac{P(a)}{P'(a)}\Big| < \frac{\eta}{3(n-1)}$$

is satisfied. Then, the Newton*-like method* (1) *is convergent, and the following is true in each iterative step*: $1^\circ \quad \zeta \in Z^{(m)}; \quad 2^\circ \quad r^{(m+1)} < \dfrac{25(n-1)}{4\eta}\,[r^{(m)}]^2$.

ON THE GRAPHS WITH MAXIMAL INDEX

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The investigation of the index of graphs is an important topic in the theory of graph spectra. The recent developments on this topic also involve the problem concerning graphs with maximal index of a given class of graphs.

Let $\mathcal{H}(n, n+t, k)$ $(t \ge -1,\ k \ge 1)$ denote the set of all connected graphs having n vertices, $n+t$ edges and k pendant vertices. A pendant vertex is a vertex of degree 1. The maximal index problem for this class has been solved by Wu, Xiao and Hong for $t = -1$, by Guo, Petrović and Gutman for $0 \le t \le 1$ and by Petrović and Borovićanin for $t = 2$. The solution of this problem for $-1 \le t \le 2$ will be presented. The maximal index problem for this class in general case is still open.

The importance of the solution for a maximal index problem in this class comes from the fact that this graph is the most irregular graph in this class (Here the proposed measure of irregularity is $\delta = \rho - \overline{d}$, where ρ denotes index and $\overline{d}$ the average degree).

SOLVING A SHORTEST PATH PROBLEM IN AN ENVIRONMENT WHICH HAS EULERIAN GRAPH REPRESENTATION WITH A FRACTAL STRUCTURE, WITH AN EMOTIONAL AGENT

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This paper presents complexity issues of solving a shortest path problem in an environment which has Eulerian graph representation with a fractal structure. The graph structure is generated from the tower of Hanoi problem. The problem is solved using emotional agent architecture.

ON A CORRELATION BETWEEN THE NATURE OF A SOLUTION OF A CLASS OF DIFFERENTIAL EQUATION OF $n-$th ORDER AND THE SOLUTIONS OF ITS ADEQUATE CHARACTERISTIC ALGEBRAIC EQUATION OF $n-$th DEGREE

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The aim of the research in this article is to find the dependence of the nature of function as a solution of a class of differential equations of n-th order with polynomial coefficients on the solutions (integers) of the adequate characteristic algebraic equation of n-th degree.

ON MATHIEU-TYPE SERIES WHICH TERMS CONTAIN GENERALIZED HYPERGEOMETRIC FUNCTION ${}_qF_p$ AND MEIJER'S G-FUNCTION

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Integral expressions and bilateral bounding inequalities are deduced for certain families of Mathieu-type series (precisely a-series and alternating a-series) in the case, when the series terms contain (i) generalized hypergeometric functions ${}_qF_p$, and (ii) Meijer's G-function. The derived closed form integral expressions and inequalities generalize some results published nowadays by the authors and H. M. Srivastava in [**1, 2**].

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ON UNICYCLIC REFLEXIVE GRAPHS

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Reflexive graphs are simple graphs whose second largest eigenvalue is at most 2. Since the spectral property that defines these graphs is hereditary, the problem of finding all reflexive graphs within the scope of a considered class is reducible to that of determining corresponding maximal graphs. Here we construct several families of maximal reflexive unicyclic graphs: some of them are deduced from certain former results on reflexive graphs, while the others are obtained directly by algebraic and combinatorial analysis supported by the aid of a computer.

ON TYPES, FORM AND SUPREMUM OF SOLUTIONS OF ORDINARY HOMOGENOUS LINEAR DIFFERENTIAL EQUATIONS OF THE SECOND ORDER

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The result has been obtained by the iterations method that if a discriminant of a homogenous linear differential equation of the second order is positive, then there are in essence no big differences between solutions, i.e. regardless of the coefficients, there are only two types of solutions.

1. Elliptical type, implying oscillatory solutions only, both limited, one converging to zero, the other remaining limited in the infinity. Those are various generalized sines and cosines depending on a single function $a(x)$, or two functions $\mathrm{Sin}_{(a,b)}(x)$, or more functions. It confirms the known presumption that every differential equation of the second order of elliptical type defines a second order trigonometry $Tg_2(a, b, \ldots, f)$; if $a = 1$ it gives classical Euclidean trigonometry of right angle.

2. Hyperbolical type, resulting mostly in monotonous solutions, but depending on coefficients it allows for finite number of zeroes within a finite range.

Very simple approximate formulae which express complex but linear oscillations by ordinary sine and cosine are given. This is the most important case in Electrical Engineering - oscillatory solutions. Some estimations of supremum of the solutions in a given interval are presented as well.

APPROXIMATE SOLUTION OF BEAM DIFFERENTIAL EQUATION IN BÉZIER FORM

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This paper investigates the use of the control points of Bézier representation for approximating the solution of the boundary value problem of the Euler-Bernouilli beam equation which is a fourth order differential equation with polynomial coefficients. Our method consists in choosing a degree n polynomial $u(t)$, then minimizing the sum of squares of Bézier control points of the residual function by Lagrange multiplier method gives the approximate solution of the beam equation. It is shown that if the problem has a unique solution, then the approximate solution obtained by this method converges to the exact solution as the degree n tends to infinity. Different boundary conditions shall be studied.

ON A CLASS OF MAXIMAL REFLEXIVE θ-GRAPHS GENERATED BY SMITH GRAPHS

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A simple graph is said to be reflexive if the second largest eigenvalue of its $(0,1)$-adjacency matrix does not exceed 2. The property $\lambda_2 \leq 2$ is a hereditary one, i.e. any induced subgraph of a reflexive graph preserves this property and that is why reflexive graphs are usually represented by maximal graphs within a given class. Bicyclic graphs whose two cycles have a common path are called θ-graphs. We consider classes of maximal reflexive θ-graphs arising from a Smith tree and a cycle attached to it in a specified way.

A LOGIC WITH IMPRECISE CONDITIONAL PROBABILITIES

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The name $LICP^S$ is an acronym for logic (L) with imprecise (I) conditional probabilities (CP) whose range is a recursive non-standard set (S). The probabilistic logic $LICP^S$ contains several types of probabilistic operators, conditional and "absolute", and

strict and imprecise. The semantics consists of KRIPKE models equipped with a probability distribution. We give here a sound and complete axiom system and prove that the problem of satisfiability of a $LICP^S$-formula is decidable. This shows that $LICP^S$ may be applied to complicated problems involving uncertain probabilistic knowledge and default reasoning.

DENSITY OF SMOOTH BOOLEAN FUNCTIONS

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The SAUER-SHELAH lemma has been instrumental in the analysis of algorithms in many areas including learning theory, combinatorial geometry, graph theory. Algorithms over discrete structures, for instance, sets of Boolean functions, often involve a search over a constrained subset which satisfies some properties. In this paper we study the complexity of classes of functions h of finite VC-dimension which satisfy a local smoothness property expressed as having long repeated values around elements of a given sample. A tight upper bound is obtained on the density of such classes. It is shown to possess a sharp threshold with respect to the smoothness parameter. Within the scope of MAGT, the results are relevant to Mathematical aspects of computer science and combinatorics.

A BIVARIATE MARSHALL AND OLKIN EXPONENTIAL MINIFICATION PROCESS

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In this paper we present a bivariate minification process with MARSHALL and OLKIN exponential distribution. The process is given by

$$X_n = K \min(X_{n-1}, Y_{n-1}, \eta_{n1}), \qquad Y_n = K \min(X_{n-1}, Y_{n-1}, \eta_{n2}),$$

where $K > \frac{\lambda}{\lambda_{12}}$, λ_1, λ_2 and λ_{12} are non-negative constants, $\lambda = \lambda_1 + \lambda_2 + \lambda_{12}$, $\{(\eta_{n1}, \eta_{n2}),$

$n \geq 1\}$ is a sequence of i.i.d. random vectors with a bivariate distribution such that (X_n, Y_n) has the bivariate MARSHALL and OLKIN exponential distribution and the random vectors (X_m, Y_m) and (η_{n1}, η_{n2}) are independent for $m < n$. The innovation distribution, the joint distribution of random vectors (X_n, Y_n) and (X_{n-j}, Y_{n-j}), $j > 0$, the autocovariance and the autocorrelation matrix are obtained. The unknown parameters are estimated and their asymptotic properties are obtained.

SPACES OF ENTIRE FUNCTIONS OF TWO COMPLEX VARIABLES

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Many studies have been made about the spaces of entire functions of one complex variables represented by either a power series or a DIRICHLET series. In this paper we study the properties of spaces of entire functions of two complex variables expressed in a double power series $f(z_1, z_2) = \sum_{m,n=0}^{\infty} a_{mn}\, z_1^m\, z_2^n,\ a_{mn} \in \mathbb{C}$. To study various properties of this space, the coefficient characterization for logarithmic type of the entire function $f(z_1, z_2)$ in terms of the coefficient $\{a_{mn}\}$ has been used. With the metric thus introduced, these spaces are found to be complete linear metric spaces. The properties of continuous linear functionals, continuous linear operators and proper bases of this space have also been obtained as necessary and sufficient conditions on the coefficients $\{a_{mn}\}$.

ON THE FRACTIONAL INTEGRALS AND DERIVATIVES IN QUANTUM CALCULUS

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For any q-integrable function $f(x)$, we define a new type of fractional q-integral $I_q^\alpha f(x)$ by

$$I_q^\alpha f(x) = \frac{x^\alpha}{\Gamma_q(\alpha)} \int_0^1 \frac{(tq;q)_\infty}{(tq^\alpha;q)_\infty} f(xt)\, \mathrm{d}_q t \qquad (\alpha \in \mathbb{R}^+),$$

where q-Pochhammer is given by $(a;q)_0 = 1\ , \quad (a;q)_n = \prod_{k=0}^{n-1} (1 - aq^k)$ $(n \in \mathbb{N} \cup \{\infty\})$, q-integral by

$$I_q(f;0,x) = \int_0^x f(t)\,\mathrm{d}_q t = x(1-q)\sum_{n=0}^{\infty} f(xq^n)\,q^n \quad (0 \le |q| \le 1),$$

and q-gamma function is defined by $\Gamma_q(z) = \frac{(q;q)_\infty}{(q^z;q)_\infty}\,(1-q)^{1-z} \quad \big(z \in \mathbb{R}\setminus\{0,-1,-2,\ldots\}\big)$.

Also, we will introduce the fractional q-derivatives.

In this paper we will justify those definitions and discuss the properties of such introduced fractional operators.

DIFFUSION-WAVE PROBLEM

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By the notion of fractional derivative of distributed order we consider the time fractional equation of distributed order

$$\int_0^2 p\,(\beta)\left[D_*^{\beta} u\,(t,\cdot)\right]\mathrm{d}\beta = \rho^2 \frac{\partial^2}{\partial x^2} u\,(t,\cdot)\,, \quad p(\beta) > 0$$

where p(β) is the weighted function, $x \in \mathbf{R}$, $t \ge 0$, subject to the initial data $u(0^+,\cdot) = \varphi(\cdot)$, $u_t(0^+,\cdot) = \psi(\cdot)$, D_*^{β} concerns the Caputo fractional derivative.

As method for construction of the solution we use in sequence Fourier and Laplace transform and their inverse.

We prove the existence-uniqueness results and regularity of the solution in the space $C^{\infty}(\mathbf{R})$, (w.r.) to the spatial variable, when the initial data are $L_2(\mathbf{R})$ functions.

We generalize result to two-term, three-term and finally n-term equation.

This equation have an application in visco-elasticity and in anomalous diffusion processes.

A CONSIDERATION OF THE SCORE SEQUENCE PAIR PROBLEMS OF (r_{11}, r_{12}, r_{22})-TOURNAMENTS

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Let G be any directed graph and S be nonnegative and non-decreasing integer sequence(s). The *prescribed degree sequence problem* is a problem to determine whether there is a graph G with S as the prescribed sequence(s) of outdegrees of the vertices [**1**, **2**]. The problem was first proposed in 1950's and many variations have been considered [**1**, **2**, **3**, **4**]. Let r_{11}, r_{12} and r_{22} be some positive integers and V be a vertex set of G. Assume that G satisfies the following (1) and (2):

(1) $V = A \cup B$ and $A \cap B = \emptyset$.

(2) For every vertex pair u, $v \in V$ $(u \neq v)$, G satisfies

$$|\{(u,\ v)\}| + |\{(v,\ u)\}| = \begin{cases} r_{11}, & \text{if } u, v \in A, \\ r_{12}, & \text{if } u \in A \text{ and } v \in B, \\ r_{22}, & \text{if } u, v \in B, \end{cases}$$

where $(u,\ v)$ ($(v,\ u)$, respectively) denotes a directed edge from u to v (from v to u).

Then G is called an $(r_{11},\ r_{12},\ r_{22})$-*tournament*. When G is an $(r_{11},\ r_{12},\ r_{22})$-tournament, the prescribed degree sequence problem is called the *score sequence pair problem of an* $(r_{11},\ r_{12},\ r_{22})$-*tournament*. The name comes from the property that each vertex of G represents a team and the direction of each directed edge indicates the win/loss of a game. Then Landau discussed the case that each team plays a game once with other teams [**3**] and we discussed the case that each team plays games r times with other teams [**4**], where r is some positive integer.

In this paper, we consider the case that there are two leagues L_1 and L_2, and that each team of L_1 (of L_2, respectively) plays games r_{11} (r_{22}) times with other teams of same league and plays games r_{12} times with teams of the other league. This case is defined as the score sequence pair problem of an $(r_{11},\ r_{12},\ r_{22})$-tournament and we propose very interesting and important discrete structures and some new type characterizations of such the problem. They lead $O(n)$ and $O(m)$ optimal algorithms of the answer of "yes/no" and of construction of the tournament easily, where $n = n_1 + n_2$ and $m = n(n-1)/2$. (The characterizations obtained by known consideration [**1**, **2**] do not lead optimal algorithm.) Furthermore, we also consider the k-edge-connectivity of the problem and obtain the similar characterizations discussed above.

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ANALYSIS OF ELECTRIC CIRCUITS WITH MATHEMATICA

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Graph theory is fully deployed in symbolic analysis of linear time-invariant electric circuits and Combinatorica, a constitutive package of Mathematica, is used to represent circuits as graphs with colored edges. Circuits are solved by using Combinatorica graph algorithms and Mathematica solvers for linear equations.

ON ONE CONJECTURE CONCERNING THE COMPLEX MANIFOLDS WITH ODD COMPLEX DIMENSION

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In this paper it is given the following conjecture: If M is an odd dimensional complex manifold, then M admits a continuous and nonsingular 1-dimensional complex subbundle of the tangent bundle. It gives sufficient conditions for a complex manifold M to admit a continuous 1-dimensional complex (2-dimensional real) subbundle of the tangent bundle. The conjecture is examined for some kinds of manifolds as complex Grassmann manifolds, flag manifolds and symmetric powers of torus. At the end it is proved that this conjecture implies that the sphere S^6 does not admit a complex structure.

STARLIKENESS AND CONVEXITY OF A CLASS OF ANALYTIC FUNCTIONS

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Let $\mathcal{A}$ be the class of analytic functions in the unit disk $\mathcal{U} = \{z : |z| < 1\}$ that are normalized with $f(0) = f'(0) - 1 = 0$ and let $-1 \leq B < A \leq 1$. Here we

study the class

$$G_{\lambda,\alpha} = \left\{ f \in \mathcal{A} : \left| \frac{1-\alpha+\alpha z f''(z)/f'(z)}{z f'(z)/f(z)} - (1-\alpha) \right| < \lambda, z \in \mathcal{U} \right\},$$

$0 \leq \alpha \leq 1$, and give sharp sufficient conditions that embed it into the classes

$$S^*[A,B] = \left\{ f \in \mathcal{A} : \frac{z f'(z)}{f(z)} \prec \frac{1+Az}{1+Bz} \right\} \text{ and } K(\delta) = \left\{ f \in \mathcal{A} : 1 + \frac{z f''(z)}{f'(z)} \prec 1+\delta z \right\},$$

where "$\prec$" denotes the usual subordination. Also, sharp upper bound of $|a_2|$ and of the Fekete-Szegö functional $|a_3 - \mu a_2^2|$ is given for the class $G_{\lambda,\alpha}$.

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PROBABILITY ON AN UNIVERSE OF DISCOURSE

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In this approach to probability, following Boole, probabilities are attributed to statements rather than events. Technically speaking, probability is defined on the Lindenbaum algebra of a propositional calculus which is a free Boolean algebra in which free generators represent stochastically independent statements (events). To these free generators we arbitrarily assign probabilities which are then defined recursively for all Boolean combinations of statements. For thus defined probability, among other things, hold the (weak) laws of large numebers of Bernoulli and Poisson and the (strong) laws of large numbers of Borel and Kolmogorov.

A COMBINATORIAL ALGORITHM FOR MAXIMUM WEIGHTED CLIQUE FOR A SUBCLASS OF PERFECT GRAPHS

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A graph G is *perfect* if every induced subgraph G' of G satisfies $\chi(G') = \omega(G')$, where χ denotes the chromatic number and ω denotes the size of a maximum clique. We say that a graph G *contains* a graph H, if H is isomorphic to an induced subgraph of G. A *hole* is a chordless cycle of length at least 4.

Berge conjectured in 1960 that a graph is perfect if and only if it contains no odd hole nor an odd antihole. This famous Strong Perfect Graph Conjecture was proved by Chudnovsky, Robertson, Seymour and Thomas in 2002. Later, Chudnovsky, Cornuéjols, Liu, Seymour and Vušković gave a polynomial time recognition algorithm for perfect graphs. In the 1980's Gröstchel, Lovàsz and Schrijver gave a polynomial time algorithm that for any perfect graph computes an optimal coloring, and a clique of maximum size. This algorithm uses the ellipsoid method and a polynomial time separation algorithm for a certain class of positive semidefinite matrices related to Lovàsz's upper bound on the Shannon capacity of a graph. The key question that remains in the area of perfect graphs is whether these optimization problems can be solved by purely combinatorial polynomial time algorithms, avoiding the numerical instability of the ellipsoid method.

We consider the class of graphs containing no odd hole, no odd antihole, and no configuration consisting of three paths between two nodes such that any two of the paths induce a hole, and at least two of the paths are of length 2. This class generalizes both claw-free perfect graphs and square-free perfect graphs. We give a combinatorial algorithm for finding a clique of maximum weight in such a graph.

NON-POLYNOMIAL LOWER BOUND FOR MONOTONE DEPTH-3 CIRCUITS COMPUTING AN NC^1-COMPLETE FUNCTION

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Valiant in [**1**] showed, by a non-probabilistic argument, an exponential lower bound for monotone $\Sigma\Pi\Sigma$-circuits computing the clique function. We prove, by a non-probabilistic argument as well, a similar result for the NC^1-complete function represented by the complete binary tree of level-alternating OR and AND gates. Our result is stronger in the sense that it applies to a much easier function than the NP-complete clique function.

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