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ON THE SEIDEL EIGENVECTORS OF A GRAPH

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Let G be a simple graph of order n and let $\mu_1 > \mu_2 > \ldots > \mu_k$ and $\mu_1^* > \mu_2^* > \ldots > \mu_k^*$ be its main eigenvalues with respect to the ordinary adjacency matrix $A = A(G)$ and the Seidel adjacency matrix $A^* = A^*(G)$, respectively. Let $n_m = n\beta_m^2$ and $n_m^* = n(\beta_m^*)^2$ for $m = 1, 2, \ldots, k$, where β_m and β_m^* are the main angles of μ_m and μ_m^*, respectively. Besides, let $(x_1^{(m)}, x_2^{(m)}, \ldots, x_n^{(m)})$ and $(s_1^{(m)}, s_2^{(m)}, \ldots, s_n^{(m)})$ denote the main eigenvectors of μ_m and μ_m^*, respectively, so that $\sum_{i=1}^n x_i^{(m)} = \sqrt{n_m}$ and $\sum_{i=1}^n s_i^{(m)} = \sqrt{n_m^*}$. In this work we show that

$$x_i^{(m)} = \left(\sum_{j=1}^{k} \frac{\sqrt{n_j^*}\ s_i^{(j)}}{\mu_j^* + 2\mu_m + 1} \right) \sqrt{n_m} \quad \text{and} \quad s_i^{(m)} = \left(\sum_{j=1}^{k} \frac{\sqrt{n_j}\ x_i^{(j)}}{\mu_m^* + 2\mu_j + 1} \right) \sqrt{n_m^*}$$

for $i = 1, 2, \ldots, n$ and $m = 1, 2, \ldots, k$.

In this paper we consider only simple graphs. The spectrum of a simple graph G of order n consists of the eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$ of its (0,1) adjacency matrix $A = A(G)$ and is denoted by $\sigma(G)$. The Seidel spectrum of G consists of the eigenvalues $\lambda_1^* \geq \lambda_2^* \geq \ldots \geq \lambda_n^*$ of its $(0, -1, 1)$ adjacency matrix $A^* = A^*(G)$ and is denoted by $\sigma^*(G)$. Let $P_G(\lambda) = |\lambda I - A|$ and $P_G^*(\lambda) = |\lambda I - A^*|$ denote the characteristic polynomial and the Seidel characteristic polynomial, respectively.

We say that an eigenvalue μ of G is main if and only if $\langle \mathbf{j}, \mathbf{P}\mathbf{j} \rangle = n \cos^2 \alpha > 0$, where $\mathbf{j}$ is the main vector (with coordinates equal to 1) and $\mathbf{P}$ is the orthogonal projection of the space $\mathbb{R}^n$ onto the eigenspace $\mathcal{E}_A(\mu)$. The quantity $\beta = |\cos\alpha|$ is called the main angle of μ. Similarly, we say that $\mu^* \in \sigma^*(G)$ is the Seidel main eigenvalue if and only if $\langle \mathbf{j}, \mathbf{P}^*\mathbf{j} \rangle = n \cos^2 \alpha^* > 0$, where $\mathbf{P}^*$ is the orthogonal projection of the space $\mathbb{R}^n$ onto the eigenspace $\mathcal{E}_{A^*}(\mu^*)$. The quantity $\beta^* = |\cos\alpha^*|$ is called the Seidel main angle of μ^*.

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Further, let $A^k = [a_{ij}^{(k)}]$ for any non-negative integer k. The number N_k of all walks of length k in G equals $\textbf{sum}\, A^k$, where $\textbf{sum}\, M$ is the sum of all elements in a matrix M. According to [**3**], [**4**], the generating function $H_G(t)$ of the numbers N_k of length k in the graph G is defined by $H_G(t) = \sum_{k=0}^{+\infty} N_k t^k$. The function $H_G^*(t) = \sum_{k=0}^{+\infty} N_k^* t^k$, where $N_k^* = \textbf{sum}\,(A^*)^k$ and $(A^*)^k = [(a_{ij}^*)^{(k)}]$, is called the Seidel generating function [**5**].

In [**1**] was proved that the graph G and its complement $\overline{G}$ have the same number of main eigenvalues. We also know that $|\mathcal{M}(G)| = |\mathcal{M}^*(G)|$, where $\mathcal{M}(G)$ and $\mathcal{M}^*(G)$ denote the sets of all main and the Seidel main eigenvalues of G, respectively (see [**2**]).

Using a procedure which is applied in [**3**,p. 45] for getting $H_G(t)$, we can easily see that

$$H_G^*\left(\frac{1}{\lambda}\right) = \lambda \left[\frac{(-1)^n 2^n P_G\left(-\dfrac{\lambda+1}{2}\right)}{P_G^*(\lambda)} - 1 \right]. \tag{1}$$

Besides, using the spectral decomposition of A^*, it is not difficult to show that $H_G^*(t)$ may be represented in the form

$$H_G^*\left(\frac{1}{\lambda}\right) = \frac{n_1^*\lambda}{\lambda-\mu_1^*} + \frac{n_2^*\lambda}{\lambda-\mu_2^*} + \ldots + \frac{n_k^*\lambda}{\lambda-\mu_k^*}, \tag{2}$$

where $n_i^* = n(\beta_i^*)^2$ and $n_1^* + n_2^* + \ldots + n_k^* = n$; μ_i^* and β_i^* $(i = 1, 2, \ldots, k)$ stand for the Seidel main eigenvalues and Seidel main angles of G, respectively. Using (1) and (2), we obtain the following relation

$$\prod_{m=1}^{k} (\lambda + 2\mu_m + 1) = \left(\prod_{m=1}^{k} (\lambda - \mu_m^*)\right)\left(1 + \sum_{m=1}^{k} \frac{n_m^*}{\lambda - \mu_m^*}\right), \tag{3}$$

where $\mu_m \in \mathcal{M}(G)$ for $m = 1, 2, \ldots, k$. We note that if $\lambda \in \sigma(G) \setminus \mathcal{M}(G)$ then necessarily $-2\lambda - 1 \in \sigma^*(G) \setminus \mathcal{M}^*(G)$. Consequently, using (3) it follows that

$$\sum_{m=1}^{k} \frac{n_m^*}{\mu_m^* + 2\mu_i + 1} = 1 \qquad (i = 1, 2, \ldots, k). \tag{4}$$

Theorem 1. *Let G be a graph of order n with two* Seidel *main eigenvalues μ_1^* and μ_2^*. Then*

$$\mu_{1,2} = \frac{n - 2 - \mu_1^* - \mu_2^*}{4} \pm \frac{\sqrt{\left(\mu_1^* - \mu_2^* + n\right)^2 - 4n_1^*\left(\mu_1^* - \mu_2^*\right)}}{4}. \tag{5}$$

Besides, we have

$$n_{1,2} = \frac{n}{2} \pm \frac{n^2 + \left(n - 2n_1^*\right)\left(\mu_1^* - \mu_2^*\right)}{2\sqrt{\left(\mu_1^* - \mu_2^* + n\right)^2 - 4n_1^*\left(\mu_1^* - \mu_2^*\right)}}, \tag{6}$$

where $n_i = n\beta_i^2$ *for* $i = 1, 2$*;* β_1 *and* β_2 *denote the main angles of* μ_1 *and* μ_2*, respectively.*

Proof. Using (3) by a straight-forward calculation we get (i) $2\mu_1 + 2\mu_2 = n - 2 - \mu_1^* - \mu_2^*$; and (ii) $4\mu_1\mu_2 = \mu_1^*\mu_2^* - (n_2^* - 1)\mu_1^* - (n_1^* - 1)\mu_2^* - (n - 1)$, which provides relation (5).

In order to derive relation (6) first recall that $A^* = K - 2A$ and $e(G) + e(\overline{G}) = \binom{n}{2}$, where K is the adjacency matrix of the complete graph K_n and $e(G)$ is the number of edges of G. Making use of (i) $2(n_1\mu_1 + n_2\mu_2) + (n_1^*\mu_1^* + n_2^*\mu_2^*) = 2\binom{n}{2}$; and (ii) $n_1 + n_2 = n$, we easily obtain (6). □

Further, according to [**3**, p. 50],

$$P_G(\lambda) = \frac{(-1)^n}{2^n} \frac{P_G^*(-2\lambda - 1)}{1 + \frac{1}{2\lambda} H_G\left(\frac{1}{\lambda}\right)} . \tag{7}$$

Setting $n_i = n\beta_i^2$ where β_i is the main angle of μ_i for $i = 1, 2, \ldots, k$, note that $H_G(t)$ can be displayed in the form

$$H_G\left(\frac{1}{\lambda}\right) = \frac{n_1\lambda}{\lambda - \mu_1} + \frac{n_2\lambda}{\lambda - \mu_2} + \ldots + \frac{n_k\lambda}{\lambda - \mu_k}.$$

Combining (7) and the last relation, it is not difficult to see that the following relation is satisfied

$$\prod_{m=1}^{k} (\lambda - \mu_m^*) = \Big(\prod_{m=1}^{k} (\lambda + 2\mu_m + 1) \Big) \Big(1 - \sum_{m=1}^{k} \frac{n_m}{\lambda + 2\mu_m + 1} \Big) , \tag{8}$$

from which we easily obtain

$$\sum_{m=1}^{k} \frac{n_m}{\mu_i^* + 2\mu_m + 1} = 1 \qquad (i = 1, 2, \ldots, k) . \tag{9}$$

Theorem 2. *Let* G *be a graph of order* n *with two main eigenvalues* μ_1 *and* μ_2*. Then*

$$\mu_{1,2}^* = \frac{n - 2 - 2\mu_1 - 2\mu_2}{2} \pm \frac{\sqrt{\big(2\mu_1 - 2\mu_2 + n\big)^2 - 8n_1\big(\mu_1 - \mu_2\big)}}{2} . \tag{10}$$

Besides, we have

$$n_{1,2}^* = \frac{n}{2} \pm \frac{n^2 + 2\big(n - 2n_1\big)\big(\mu_1 - \mu_2\big)}{2\sqrt{\big(2\mu_1 - 2\mu_2 + n\big)^2 - 8n_1\big(\mu_1 - \mu_2\big)}} , \tag{11}$$

where $n_1^* = n(\beta_1^*)^2$ *and* $n_2^* = n(\beta_2^*)^2$*.*

Proof. Using (8) by an easy calculation we have (i) $\mu_1^* + \mu_2^* = n - 2 - 2\mu_1 - 2\mu_2$; and (ii) $\mu_1^*\mu_2^* = 4\mu_1\mu_2 - 2(n_2-1)\mu_1 - 2(n_1-1)\mu_2 - (n-1)$, which provides relation (10).

Next, making use of (i) $(n_1^*\mu_1^* + n_2^*\mu_2^*) + 2(n_1\mu_1 + n_2\mu_2) = 2\binom{n}{2}$ and (ii) $n_1^* + n_2^* = n$, we arrive at (11), which completes the proof. □

Let $(x_1^{(m)}, x_2^{(m)}, \ldots, x_n^{(m)})$ and $(s_1^{(m)}, s_2^{(m)}, \ldots, s_n^{(m)})$ denote the main eigenvectors of μ_m and μ_m^*, respectively, so that $\sum_{i=1}^n x_i^{(m)} = \sqrt{n_m}$ and $\sum_{i=1}^n s_i^{(m)} = \sqrt{n_m^*}$. Let $\deg(i)$ denotes the degree of the vertex i. With this notation, we have recently proved the following result:

Proposition 1 (Lepović [**6**]). *Let* $\mu_1, \mu_2, \ldots, \mu_k \in \mathcal{M}(G)$. *Then*:

(1°) $\sqrt{n_1}\, x_i^{(1)} + \sqrt{n_2}\, x_i^{(2)} + \ldots + \sqrt{n_k}\, x_i^{(k)} = 1;$

(2°) $\sqrt{n_1}\, x_i^{(1)}\mu_1 + \sqrt{n_2}\, x_i^{(2)}\mu_2 + \ldots + \sqrt{n_k}\, x_i^{(k)}\mu_k = \deg(i),$

for any $i = 1, 2, \ldots, n$.

Proposition 2. *Let* $\mu_1^*, \mu_2^*, \ldots, \mu_k^* \in \mathcal{M}^*(G)$. *Then*:

(1°) $\sqrt{n_1^*}\, s_i^{(1)} + \sqrt{n_2^*}\, s_i^{(2)} + \ldots + \sqrt{n_k^*}\, s_i^{(k)} = 1;$

(2°) $\sqrt{n_1^*}\, s_i^{(1)}\mu_1^* + \sqrt{n_2^*}\, s_i^{(2)}\mu_2^* + \ldots + \sqrt{n_k^*}\, s_i^{(k)}\mu_k^* = (n-1) - 2\deg(i)\,,$

for any $i = 1, 2, \ldots, n$.

Proof. Let $\mathbb{M} = \{\, m \mid \lambda_m^* \in \sigma^*(G) \setminus \mathcal{M}^*(G)\}$. Then for any non-negative integer ℓ, we have

$$(A^*)^\ell = \sum_{m=1}^{k} \mathbf{X}_m^* \,(\mu_m^*)^\ell + \sum_{m\in\mathbb{M}} \mathbf{Y}_m \,(\lambda_m^*)^\ell\,,$$

where $\mathbf{X}_m^* = \mathbf{X}_m^*[s_{ij}^{(m)}]$ and $\mathbf{Y}_m = \mathbf{Y}_m[\,y_{ij}^{(m)}]$ with $z_{ij}^{(m)} = z_i^{(m)} z_j^{(m)}$. Of course, here $\{\,(y_1^{(m)}, y_2^{(m)}, \ldots, y_n^{(m)}) \mid m \in \mathbb{M}\,\}$ represents a complete set of mutually orthogonal normalized eigenvectors which are related to $\{\lambda_m^* \in \sigma^*(G) \setminus \mathcal{M}^*(G) \mid m \in \mathbb{M}\,\}$ such that $y_1^{(m)} + y_2^{(m)} + \ldots + y_n^{(m)} = 0$. Using the last relation we obtain

$$\sum_{j=1}^{n} (a_{ij}^*)^{(\ell)} = \sum_{m=1}^{k} \Big(\sum_{j=1}^{n} s_j^{(m)}\Big) s_i^{(m)} (\mu_m^*)^\ell + \sum_{m\in\mathbb{M}} \Big(\sum_{j=1}^{n} y_j^{(m)}\Big) y_i^{(m)} (\lambda_m^*)^\ell\,,$$

which provides the proof. □

Proposition 3. *Let G be a connected or disconnected graph of order n with two Seidel main eigenvalues μ_1^* and μ_2^*. Then*

$$s_i^{(1)} = \frac{\big((n-1) - 2\deg(i)\big) - \mu_2^*}{\sqrt{n_1^*}\,(\,\mu_1^* - \mu_2^*)} \quad \text{and} \quad s_i^{(2)} = \frac{\mu_1^* - \big((n-1) - 2\deg(i)\big)}{\sqrt{n_2^*}\,(\,\mu_1^* - \mu_2^*)}$$

for $i = 1, 2, \ldots, n$.

Proof. Combing relations Proposition 2 (1°) and (2°), by a straight-forward calculation we obtain the statement. □

Remark 1. Using Proposition 1 (1°) and (2°), we easily find that (i) $x_i^{(1)} = \dfrac{\deg(i)-\mu_2}{\sqrt{n_1}\,(\mu_1-\mu_2)}$ and (ii) $x_i^{(2)} = \dfrac{\mu_1-\deg(i)}{\sqrt{n_2}\,(\mu_1-\mu_2)}$ for $i=1,2,\ldots,n$.

Theorem 3. *Let G be a connected or disconnected graph of order n with k main eigenvalues. Let $(x_1^{(m)},x_2^{(m)},\ldots,x_n^{(m)})$ and $(s_1^{(m)},s_2^{(m)},\ldots,s_n^{(m)})$ denote the main eigenvectors of μ_m and μ_m^*, respectively, so that $\sum_{i=1}^{n} x_i^{(m)} = \sqrt{n_m}$ and $\sum_{i=1}^{n} s_i^{(m)} = \sqrt{n_m^*}$. Then*

$$(12)\quad x_i^{(m)} = \Big(\sum_{j=1}^{k} \frac{\sqrt{n_j^*}\; s_i^{(j)}}{\mu_j^*+2\mu_m+1}\Big)\sqrt{n_m} \quad \text{and} \quad s_i^{(m)} = \Big(\sum_{j=1}^{k} \frac{\sqrt{n_j}\; x_i^{(j)}}{\mu_m^*+2\mu_j+1}\Big)\sqrt{n_m^*}$$

for $i=1,2,\ldots,n$ and $m=1,2,\ldots,k$.

Proof. We shall first show that the right-hand side of (12) is true for any $i=1,2,\ldots,n$ and $m=1,2,\ldots,k$. In order to prove that $(s_1^{(m)},s_2^{(m)},\ldots,s_n^{(m)})$ is the eigenvector of the main eigenvalue μ_m^*, it is sufficient to demonstrate that its coordinates $s_i^{(m)}$ satisfy the following system of homogeneous linear equations

$$\mu_m^*\, s_i^{(m)} = \sum_{\ell=1}^{n}\Big(\sum_{j=1}^{k}\big[(1-2a_{i\ell})-a_{i\ell}^{(0)}\big]\,\frac{\sqrt{n_j}\; x_\ell^{(j)}}{\mu_m^*+2\mu_j+1}\Big)\sqrt{n_m^*},$$

understanding that $a_{ij}^{(0)} = \delta_{ij}$, where δ_{ij} is the Kronecker delta symbol. Indeed, using (9) and the equation $\mu_j\, x_i^{(j)} = \sum_{\ell=1}^{n} a_{i\ell}\, x_\ell^{(j)}$, the last relation is transformed into

$$\mu_m^*\, s_i^{(m)} = \sqrt{n_m^*} - \Big(\sum_{j=1}^{k}\frac{\sqrt{n_j}\, x_i^{(j)}\,(2\mu_j+1)}{\mu_m^*+2\mu_j+1}\Big)\sqrt{n_m^*}\,,$$

from which we obtain the assertion using Proposition 1 (1°). In a quite analogous manner, making use of (4) and Proposition 2 (1°), we find that the left-hand side of (12) is also true for any $i=1,2,\ldots,n$ and $m=1,2,\ldots,k$. □

Using (12) and keeping in mind that $\{(x_1^{(m)},x_2^{(m)},\ldots,x_n^{(m)})\,|\,m=1,2,\ldots,k\}$ and $\{(s_1^{(m)},s_2^{(m)},\ldots,s_n^{(m)})\,|\,m=1,2,\ldots,k\}$ are the complete systems of mutually orthogonal normalized eigenvectors, we arrive at

Proposition 4. *Let G be a graph with k main eigenvalues. Then for any $m=1,2,\ldots,k$ we have*

$$\frac{1}{n_m} = \sum_{i=1}^{k}\frac{n_i^*}{\big(\mu_i^*+2\mu_m+1\big)^2} \quad \textit{and} \quad \frac{1}{n_m^*} = \sum_{i=1}^{k}\frac{n_i}{\big(\mu_m^*+2\mu_i+1\big)^2}\,.$$

Further, for any $\lambda^* \in \sigma^*(G)$ we have that $-\lambda^* \in \sigma(\overline{G})$. Since $\mathcal{E}_{A^*}(\lambda^*) = \mathcal{E}_{\overline{A}^*}(-\lambda^*)$, we obtain implicitly $\mathcal{M}^*(\overline{G}) = -\mathcal{M}^*(G)$, where $-\mathcal{M}^*(G) = \{\lambda^* \mid -\lambda^* \in \mathcal{M}^*(G)\}$. Consequently, according to (2), we get

$$H_{\overline{G}^*}\Big(\frac{1}{\lambda}\Big) = \frac{n_1^*\lambda}{\lambda+\mu_1^*} + \frac{n_2^*\lambda}{\lambda+\mu_2^*} + \dots + \frac{n_k^*\lambda}{\lambda+\mu_k^*}\,. \tag{13}$$

Next, let $(\overline{x}_1^{(m)}, \overline{x}_2^{(m)}, \dots, \overline{x}_n^{(m)})$ denote the main eigenvector of $\overline{\mu}_m \in \mathcal{M}(\overline{G})$ such that $\sum_{i=1}^n \overline{x}_i^{(m)} = \sqrt{\overline{n}_m}$, where $\overline{n}_m = n\overline{\beta}_m^{\,2}$ and $\overline{\beta}_m$ is the main angle of $\overline{\mu}_m$. Using the last relation, we obtain from Theorem 3 that

$$\overline{x}_i^{(m)} = \Big(\sum_{j=1}^k \frac{\sqrt{n_j^*}\, s_i^{(j)}}{-\mu_j^* + 2\overline{\mu}_m + 1}\Big)\sqrt{\overline{n}_m} \quad \text{and} \quad s_i^{(m)} = \Big(\sum_{j=1}^k \frac{\sqrt{\overline{n}_j}\, \overline{x}_i^{(j)}}{-\mu_m^* + 2\overline{\mu}_j + 1}\Big)\sqrt{n_m^*},$$

and from Proposition 4 that

$$\frac{1}{\overline{n}_m} = \sum_{i=1}^k \frac{n_i^*}{\big(-\mu_i^* + 2\overline{\mu}_m + 1\big)^2} \quad \text{and} \quad \frac{1}{n_m^*} = \sum_{i=1}^k \frac{\overline{n}_i}{\big(-\mu_m^* + 2\overline{\mu}_i + 1\big)^2}\,.$$

We note that if G is a self-complementary graph then its SEIDEL main spectrum is symmetric with respect to the zero point. Moreover, according to (2) and (13) it follows that $n_{\mu^*} = n_{-\mu^*}$ for any $\mu^*, -\mu^* \in \mathcal{M}^*(G)$, by understanding that $n_{\mu^*} = n\beta_{\mu^*}^2$ and β_{μ^*} is the main angle of μ^*. In particular, if G is a self-complementary graph of order n with two SEIDEL main eigenvalues μ^* and $-\mu^*$, we obtain implicitly from Proposition 3 that $\pm\mu^* = \pm\sqrt{\frac{1}{n}\sum_{i=1}^n \big((n-1) - 2\deg(i)\big)^2}$. We also note that in this case $n_{\mu^*} = n/2$. Thus, if G is a self-complementary graph of order n with two main eigenvalues μ_1 and μ_2, we find from (5) and (6) that

$$\mu_{1,2} = \frac{n-2}{4} \pm \frac{\sqrt{n^2 + 4(\mu^*)^2}}{4} \quad \text{and} \quad n_{1,2} = \frac{n}{2} \pm \frac{n^2}{2\sqrt{n^2 + 4(\mu^*)^2}}\,,$$

respectively.

Finally, in order to demonstrate some results presented in this paper we shall consider the following graph: $G = K_{2n+1} \cup 3(n+1)K_1$, where nK_1 denotes a graph with n isolated vertices. We can see that $K_{2n+1} \cup 3(n+1)K_1$ is integral[1] in the ordinary and the SEIDEL sense for any non-negative integer n. Namely, $\sigma(G)$ and $\sigma^*(G)$ have two main eigenvalues $\mu_1 = 2n$, $\mu_2 = 0$ and two SEIDEL main eigenvalues $\mu_1^* = 4n+3$, $\mu_2^* = -(3n+1)$, respectively. Further, we have that (i) $n_1 = 2n+1$ and $n_2 = 3(n+1)$; (ii) $n_1^* = \dfrac{32n^2+48n+16}{7n+4}$ and $n_2^* = \dfrac{3n^2}{7n+4}$;

[1]A graph is called integral if its spectrum consists entirely of integers. We notice from (8) that if G is integral then G is integral in the SEIDEL sense if and only if the SEIDEL main spectrum $\mathcal{M}^*(G)$ contains integral values.

(iii) $\sigma(G) = \{2n, 0^{3(n+1)}, -1^{2n}\}$ and $\sigma^*(G) = \{4n+3, 1^{2n}, -1^{3n+2}, -(3n+1)\}$, where the multiplicity of a multiple eigenvalue is given in the form of an exponent. Besides, we find that (iv) $x_i^{(1)} = \dfrac{1}{\sqrt{2n+1}}$ if $i \in V(K_{2n+1})$ and $x_i^{(1)} = 0$, otherwise, where $V(G)$ is the vertex set of G; (v) $x_i^{(2)} = 0$ if $i \in V(K_{2n+1})$ and $x_i^{(2)} = \dfrac{1}{\sqrt{3(n+1)}}$, otherwise; (vi) $s_i^{(1)} = \dfrac{\sqrt{n+1}}{\sqrt{(2n+1)(7n+4)}}$ if $i \in V(K_{2n+1})$ and $s_i^{(1)} = \dfrac{\sqrt{2n+1}}{\sqrt{(n+1)(7n+4)}}$, otherwise; and (vii) $s_i^{(2)} = \dfrac{\sqrt{3}}{\sqrt{7n+4}}$ if $i \in V(K_{2n+1})$ and $s_i^{(2)} = \dfrac{-\sqrt{3}}{3\sqrt{7n+4}}$, otherwise.

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