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A SIMPLIFICATION OF THE PAPER OF KOHNEN

Nenad P. Cakić

In this short note we indicate that a recurrence from the paper [1] is well-known.

In the recent paper [**1**], W. Kohnen analyzed the following recurrence:

$$a_n = \left(1 - \frac{1}{n}\right) a_{n-1} + \frac{1}{n}\, a_{n-2} \tag{1}$$

for $n \geq 2$ and $a_0 = 1$, $a_1 = 0$.

However, such recursions were itensively studied. Namely, if we introduce the substitution $n!\,a_n = d_n$, we get

$$d_n = (n-1)(d_{n-1} + d_{n-2}), \tag{2}$$

with $d_0 = 1$, $d_1 = 0$, which is a well-known recurrence for the number of derangements of the set $[n]$ (see, for example the "old" references [**2**], [**3**] or or the "new" reference [**4**]).

So, the fact that $a_n = \displaystyle\sum_{\nu=0}^{n} \frac{(-1)^\nu}{\nu!}$ follows immediately from equality $d_n = n! \displaystyle\sum_{\nu=0}^{n} \frac{(-1)^\nu}{\nu!}$, and does not really require a proof!

Remark 1. A generalized problem on the line of d_n is the recurrence $f(n+1) = n\big(f(n) + f(n-1)\big)$ with initial conditions $f(0) = a,\ f(1) = b$, and with the solution ([**5**]):

$$f(n) = (a-b)\,n! \sum_{k=0}^{n} \frac{(-1)^k}{k!} + b\,n!.$$

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Putting $a = 1$ and $b = 0$ we find $f(n) = n! \sum_{k=0}^{n} \frac{(-1)^\nu}{\nu!}$.

REMARK 2. In [**2**], pp. 201, we can find another expression for d_n :

$$d_n = \sum_{r=0}^{n-1} (-1)^r \binom{n}{r} (n-r)^r (n-r-1)^{n-r}.$$

REFERENCES

1. W. KOHNEN: *A remark on the left - factorial hypothesis.* Publ. Elektroteh. Fak. Ser. Mat. **9** (1998), 51-54.
2. L. COMTET: *Advanced combinatorics.* Reidel, Dodrecht, 1974.
3. D. E. KNUTH: *The art of computer programing.* Vol. 3, Addison - Wesley, Rading MA, 1973.
4. H. S. WILF: *Generatingfunctionology.* (2nd ed.) Academic Press, San Diego, 1994.
5. N. P. CAKIĆ: *Solution of the problem* B - 853. The Fibomacci Quarterly (to appear).

University of Niš,
Faculty of Technology,
Department of Mathematics,
16000 Leskovac, Yugoslavia
ecakic@ubbg.etf.bg.ac.yu

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