UNIV. BEOGRAD. PUBL. ELEKTROTEHN. FAK.
Ser. Mat. 7 (1996), 72–73.

EXTENSIONS OF AN INEQUALITY

Murray S. Klamkin

Dedicated to the memory of Professor Dragoslav S. Mitrinović

Inequality (2) is proved.

The inequality

$$a_1 a_2 \cdots a_n \geq \Big(S-(n-1)a_1\Big)\Big(S-(n-1)a_2\Big)\cdots\Big(S-(n-1)a_n\Big), \tag{1}$$

$S = a_1 + a_2 + \cdots + a_n$, $a_i > 0$ and all the factors in the right hand side are nonnegative, was established by MITRINOVIĆ and ADAMOVIĆ [**1**] and an extension of it was given in [**2**] (there is no loss of generality by assuming all the right hand factors are positive). Here we give a generalization of (1) which leads to a different extension.

We may assume that $a_1 \geq a_2 \cdots \geq a_n$. It is now easy to show that the vector

$$\Big(S-(n-1)a_n, S-(n-1)a_{n-1}, \ldots, S-(n-1)a_1\Big),$$

majorizes the vector $(a_1, a_2, \ldots, a_n)$, i.e.,

$$S-(n-1)a_n + S-(n-1)a_{n-1} + \cdots + S-(n-1)a_{n-r+1} \geq a_1 + a_2 + \cdots + a_r$$

for $r = 1, 2, \ldots, n-1$, and equality for $r = n$. It now follows by the majorization inequality [**3**] that

$$\sum_{i=1}^{n} F\big(S-(n-1)a_i\big) \geq \sum_{i=1}^{n} F(a_i) \tag{2}$$

for all convex functions over the given domain. In particular, by choosing $F(x) = -\ln x$, we obtain (1). Letting $F(x) = x^p$, we obtain

[0] 1991 Mathematics Subject Classification: 26D15

$$(S-(n-1)a_1)^p + (S-(n-1)a_2)^p + \cdots + (S-(n-1)a_n)^p \geq {a_1}^p + {a_2}^p + \cdots + {a_n}^p \tag{3}$$

for $p > 1$ or $p < 0$. For $1 > p > 0$, the inequality is reversed. For p a positive even integer, x^p is convex for all x so that in this case the a_i and the $S-(n-1)a_i$ need not all be positive.

Finaly, we can generate many triangle inequalities from (2). For example, since x^x and $x \ln x$ are convex for $x > 0$,

$$(b+c-a)^{b+c-a} + (c+a-b)^{c+a-b} + (a+b-c)^{a+b-c} \geq a^a + b^b + c^c, \tag{4}$$

$$(b+c-a)^{b+c-a}(c+a-b)^{c+a-b}(a+b-c)^{a+b-c} \geq a^a b^b c^c, \tag{5}$$

where a, b, c are sides of a triangle.

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Department of Mathematics,
University of Alberta,
Edmonton,
Canada T6G 2G1

(Received September 25, 1995)