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717. VARIATIONS AND GENERALISATIONS OF CLAIRAUT'S EQUATIONS*

D. S. Mitrinović and *J. D. Kečkić*

Clairaut's differential equation (1.1) was generalised a number of times in various directions. This is an up to date research expository paper presenting a historical survey of those generalisations to first and higher order ordinary and partial differential equations, as well as to difference equations.

1. Introduction. Differential equation

$$y = xy' + f(y') \tag{1.1}$$

is called CLAIRAUT's equation (ALEXIS CLAUDE CLAIRAUT, 1713—1765). As known, the general solution of (1.1) is

$$y = Cx + f(C) \qquad (C \text{ arbitrary constant}) \tag{1.2}$$

and there exists a singular solution which, in parametric form, reads

$$y = xt + f(t), \qquad 0 = x + f'(t). \tag{1.3}$$

Geometrically, the singular solution (1.3) is the envelope of the family of straight lines (1.2).

Partial differential equation

$$u = xu_x + yu_y + f(u_x,\ u_y) \tag{1.4}$$

is also called CLAIRAUT's equation. The complete integral of (1.4) reads

$$u = C_1 x + C_2 y + f(C_1,\ C_2) \qquad (C_1,\ C_2 \text{ arbitrary constants}). \tag{1.5}$$

The interesting form of the general solution (1.2) of (1.1) namely, the derivative is simply replaced by an arbitrary constant — attracted the attention of a number of mathematicians, who mainly tried to form more general equa-

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tions with analogous property. The research was carried out mainly in the following directions:

(i) Formation of the most general first order differential equation whose general solution is obtained by replacing the derivative by an arbitrary constant;

(ii) Formation of higher order equations whose general solutions are obtained by replacing the derivatives by arbitrary constants;

(iii) Formation of partial differential equations whose complete integrals have the described property;

(iv) Investigation of singular solutions;

(v) Formation of difference equations with analogous properties.

In further text the letter C, with or without indices, will always denote an arbitrary constant.

2. First order equations. The first paper we found dates from 1847. GASPARIS [1] noticed that „the general integral of the equation

$$F(y-xy',\ y')=0$$

is obtained in the same way as the integral of CLAIRAUT's equation". The partial differential equation

$$F(u-xu_x-yu_y,\ u_x,\ u_y)=0$$

is also considered in [1], but its complete integral is not explicitly mentioned.

P. MANSION [2] investigated which first order equations have the property of CLAIRAUT's equation, and concluded: „CLAIRAUT's equation (1.1) is the only equation of the second order whose integral is obtained by replacing y' by an arbitrary constant". Similarly, for the equation (1.4) MANSION states „this equation alone among equations of the first order enjoys the property of having for its complete integral the same equation with u_x and u_y replaced by constants".

Both statements, as they stand, are not correct. Indeed, as noted by GOURSAT [3], the equation

$$y=y'+\frac{e^x}{y'} \tag{2.1}$$

which is not of CLAIRAUT's type, has the general solution

$$y=C+\frac{e^x}{C}. \tag{2.2}$$

We add that, similarly, the partial differential equation

$$u=u_x+u_y+\frac{e^x}{u_x}+\frac{e^y}{u_y} \tag{2.3}$$

which is not of type (1.4), has the complete integral

$$u=C_1+C_2+\frac{e^x}{C_1}+\frac{e^y}{C_2}.$$

REMARK. It is interesting to note MANSION's slip when he called (1.1) „equation of the second order“.

In fact, if we start with the equation $y'=f(x, y)$, where f is a *regular* function in the neighbourhood of a point (x_0, y_0), and we request that $f(x, y)=C$ is the general solution of that equations, we can conclude that only CLAIRAUT's equation has that property. The equation (2.1), however, does not belong to that class. A similar comment holds for the partial differential equation (2.3).

RAFFY [4] made the first serious contribution to the problem of widening the class of equations with CLAIRAUT's property. He proved the following result:

Any differential equation of the form

$$y=F(f(y'))+F(x-f(y')),$$

where F' and f are inverse functions, has the general solution

$$y=F(C)+F(x-C),$$

i.e. it is enough to replace the derivative by an arbitrary constant. We quote some examples from RAFFY's paper [4]:

(i) $F(t)=e^t$ leads to the equation (2.1) and its general solution (2.2);

(ii) $F(t)=\log t$ leads to the equation

$$y=\log\frac{1}{y'}+\log\left(x-\frac{1}{y'}\right)$$

with the general solution $y=\log\frac{1}{C}+\log\left(x-\frac{1}{C}\right)$;

(iii) $f(t)=-\operatorname{arctg} t$ leads to the equation

$$y=\log\frac{\cos x-y'\sin x}{1+(y')^2}$$

with the general solution $y=\log\frac{\cos x-C\sin x}{1+C^2}$.

RAFFY also tried to form *all* the equations of the form $y=G(x, y')$ which have the property of CLAIRAUT's equation. The problem of determining the function G RAFFY reduced to the functional-differential equation

$$G\left(x, \frac{\partial G(x, t)}{\partial x}\right)=G(x, t). \tag{2.4}$$

The equation (2.4) clearly has the following solutions:

$$G(x, t)=xt+f(t) \qquad (f \text{ arbitrary function});$$

$$G(x, t)=F(f(t))+F(x-f(t)) \qquad (F' \text{ and } f \text{ inverse functions}),$$

but there are probably other solutions of (2.4).

Continuing RAFFY's investigations, GOURSAT in the mentioned paper [3] gave a procedure for forming all the equations of the form

$$(y')^2 + P(x, y)\, y' + Q(x, y) = 0,$$

whose general solution is given by

$$C^2 + P(x, y)\, C + Q(x, y) = 0.$$

LEVY's paper [5] is interesting. By geometrical arguments he examined differential equations of the form

$$F(x, y, y') = 0 \tag{2.5}$$

and concluded that if the condition

$$\frac{\partial F}{\partial x} + y' \frac{\partial F}{\partial y} = 0 \tag{2.6}$$

is fulfilled, then (2.5) has the property of CLAIRAUT'S equation, and, what is more, can be reduced to CLAIRAUT's equation.

The condition (2.6) is a useful criterion for testing whether (2.5) is a CLAIRAUT type equation, since, in general, it is not possible to solve (2.5) with respect to y.

LEVY also gave examples of equations (2.5) which do not fulfil (2.6), but whose general solution is obtained by replacing y' by C. Two such examples are:

$$(y')^2 - xy' + 2x^2 - y = 0, \qquad (y')^2 - 2xy' - e^{2x} = 0.$$

3. Higher order equations. After his result for first order equations, RAFFY began to examine higher order equations with analogous property. His first result from short paper [6] deals with linear equations. Namely, RAFFY proved the following:

If an n-th order linear differential equations has particular integrals x, $x^2, \ldots, x^n$, then its general solution is formed by replacing the derivatives by arbitrary constants.

This theorem is proved by constructing the equation whose particular integrals are $x, x^2, \ldots, x^n$. The equation reads

$$y = xy' - \frac{1}{2!} x^2 y'' + \cdots + \frac{(-1)^{n-1}}{n!} x^n y^{(n)},$$

and its general solution is

$$y = C_1 x + C_2 x^2 + \cdots + C_n x^n.$$

RAFFY then states that the above equation shows that there exist differential equations of arbitrary order whose general solutions are formed by replacing the derivatives by arbitrary constants. „It would be interesting, but doubtless hard, to form all such equations", comments RAFFY at the end of [6].

In his next paper [7] RAFFY proved the result:

The general solution of the equation

$$(3.1)\qquad y-xy'+\frac{1}{2!}x^2y''-\cdots+\frac{(-1)^n}{n!}x^ny^{(n)}=F\left(\frac{\partial\varphi}{\partial x},\frac{\partial^2\varphi}{\partial x^2},\ldots,\frac{\partial^n\varphi}{\partial x^n}\right),$$

where $\varphi\equiv y-xy'+\frac{1}{2!}x^2y''-\cdots+\frac{(-1)^n}{n!}x^ny^{(n)}$, and where $\frac{\partial\varphi}{\partial x}$ denotes differentiation with respect to x; $y, y', \ldots, y^{(n)}$ being treated as constants, is given by

$$(3.2)\qquad y+C_1x-\frac{1}{2!}C_2x^2+\cdots-\frac{(-1)^n}{n!}C_nx^n=F(C_1,C_2,\ldots,C_n).$$

Though the equation (3.1) and its general solution (3.2) are generalisations of CLAIRAUT's equation (1.1) and its general solution (1.2), since for $n=1$ (3.1) reduces to (1.1) and (3.2) to (1.2), this result is not an answer to the problem RAFFY posed in [6]. Indeed, the general solution of (3.1) is not obtained by replacing the derivatives by arbitrary constants.

Starting with RAFFY's result from [6], CHINI [8] determined *all* the linear second order equations whose general solution is formed when the derivatives are replaced by arbitrary constants. Those are equations of the form

$$y=(x+k)y'+f(x)y'',$$

where f satisfies $ff''+(x+k)f'=f$, and k is a constant, but f is not a linear function of x; and

$$y=ay'+\frac{a(1-a')}{a''}y'',$$

where a satisfies $a'''=(a'')^2(Ka^{-2}+(a'-1)^{-1})$, and K is a nonzero constant.

A special case of equation (3.1), namely the equation

$$(3.3)\qquad y=xy'-\frac{1}{2!}x^2y''+\cdots+\frac{(-1)^{n-1}}{n!}x^ny^{(n)}+G(y^{(n)})$$

was solved again by WITTY [9]. KLAMKIN [10] pointed out that the equation

$$(3.4)\qquad F\left(y-xy'+\frac{1}{2}x^2y'',\ y'-xy'',\ y''\right)=0$$

was given as a problem in [11]. He also solved the general equation (3.1). Clearly, both authors were unaware of RAFFY's result.

REMARK. The equation (3.4) together with its general solution is also recorded in [12].

Finally, we mention CUNNINGHAM's interesting paper [13]. He introduced the notion of CLAIRAUT's functions, or Clairautians ${}^kU_{r,\,n}$, which he defined by

$$ {}^kU_{r,\,n}=\sum_{\nu=0}^{r}\frac{(k+r-\nu)!(-1)^{\nu}}{k!\,(n-\nu)!\,\nu!}\,x^{\nu}\,y^{(n-r+\nu)}, $$

and he called equations which involve Clairautians CLAIRAUT's equations. The following properties of Clairautians were established:

$$ {}^kU_{0,\,n}=y^{(n)};\qquad {}^0U_{r,\,n}=\frac{(-1)^r}{r!}\,x^n\,y^{(n)}; $$

$$ {}^kU_{r,\,n}=\sum_{\nu=0}^{r}{}^{k-1}U_{\nu,\,n-r+\nu};\qquad {}^kU_{r,\,n}-{}^{k-1}U_{r,\,n}={}^kU_{r-1,\,n-1}, $$

$$ \frac{\mathrm{d}^{\nu}}{\mathrm{d}x^{\nu}}\,{}^kU_{r,\,n}={}^{k-\nu}U_{r,\,n+\nu};\qquad {}^kU_{r,\,n}=\frac{(-1)^r}{r!}\,x^{k+r}\,\frac{\mathrm{d}^r}{\mathrm{d}x^r}\,x^{-k}\,y^{(n-r)}. $$

CUNNINGHAM states that using the above properties solutions of many CLAIRAUT's equations „may be founded and effected with elegance", but he did not do that in [13].

4. Singular solutions. Singular solutions of the equation (3.1) were considered in detail by BOUNITZKY [14]. He proved the following result:

Singular solutions of the equation (3.1) are the solutions of the system consisting of (3.1) and the equation

$$ \frac{(-1)^n}{n!}\,x^n-\frac{(-1)^{n-1}}{(n-1)!}\,\frac{\partial F}{\partial\varphi_1}\,x^{n-1}-\cdots-\frac{\partial F}{\partial\varphi_n}=0, $$

where $\varphi_k=(-1)^k\,\partial^k\varphi/\partial x^k$.

Singular solutions are, geometrically, those envelopes of certain families of particular integrals of (3.1) which have n-th order contact with each member of the family.

The equation (3.1) need not have singular solutions, as for example in the case of the (linear) equation

$$ y-xy'+\frac{x^2}{2}\,y''-y''=0, $$

but if it exists, if will, in general, contain $n-1$ arbitrary constants.

BOUNITZKY paid special attention to the equation (3.3) and found that its singular solution is obtained by the elimination of t between the equations

$$(4.1)\qquad \begin{aligned} &y=\int_{\alpha}^{t}\left(x+(n!\,G'(n!\,t))^{1/n}\right)^{n}dt+ax^{n}+G(n!\,t)+b_1x+\cdots+b_{n-1}x^{n-1},\\ &x+(n!\,G(n!\,t))^{1/n}=0.\end{aligned}$$

REMARK. RAFFY in his letter [15] states that the results from [14] were previously published by DIXON [16], and hence that the priority belongs to DIXON. In fact, in paper [16] DIXON gives a general theory of singular solutions for systems of differential equations, so that BOUNITZKY's results are formally special cases of DIXON's. However, since BOUNITZKY examined a fixed class of equations, he arrived at some special results which hold only for that class. Therefore, BOUNITZKY's results are not simple special cases of DIXON's results.

Unaware of the paper [14], KEČKIĆ [17] studied again the singular solutions of the equation (3.3), and he arrived at a formula which is much simpler then BOUNITZKY's formula (4.1). Namely, in [17] it is stated:

The singular solution of (3.3) is given by

$$y=Y+\sum_{k=1}^{n-1}C_k x^k,$$

where Y is a solution of the equation $G'(y^{(n)})=\frac{(-1)^n}{n!}x^n$.

The geometrical relationship between the general and the singular solution is also studied in [17], but only for the equation

$$y=xy'-\frac{1}{2}x^2y''+(y'')^2.$$

5. Partial differential equations. As we already mentioned, in paper [1] one can find a slight generalisation of CLAIRAUT's equation (1.4), though its complete integral is not given explicitely.

The „linear part" of CLAIRAUT's equation (1.4), i.e. the equation $u=xu_x+yu_y$ was generalised by MITRINOVIĆ [18], who noted that the complete integral of the equation

$$(5.1)\qquad u=xu_x+yu_y-\frac{1}{2!}(xu_x+yu_y)^{(2)}+\cdots+\frac{(-1)^{n-1}}{n!}(xu_x+yu_y)^{(n)}.$$

where

$$(xu_x+yu_y)^{(k)}=\sum_{\nu=0}^{k}\binom{k}{\nu}x^{k-\nu}y^{\nu}\frac{\partial^k u}{\partial x^{k-\nu}\partial y^{\nu}}$$

is obtained when each derivative in (5.1) is replaced by an arbitrary constant. This result was proved in [19].

In paper [20] the following result is given:
The complete integral of the equation

$$u = xu_x + yu_y - \frac{1}{2!}(xu_x + yu_y)^{(2)} + \cdots + (-1)^{n-1}\frac{1}{n!}(xu_x + yu_y)^{(n)}$$
$$+ F\left(\frac{\partial^n u}{\partial x^n}, \frac{\partial^n u}{\partial x^{n-1}\partial y}, \ldots, \frac{\partial^n u}{\partial y^n}\right)$$

is

$$u = \sum_{k=1}^{n}\frac{1}{k!}\sum_{\nu=0}^{k}\binom{k}{\nu} C_k^\nu x^{k-\nu} y^\nu + F(C_n^0, C_n^1, \ldots, C_n^n).$$

CLAIRAUT's partial differential equations where the unknown function dependes on more than two variables are also mentioned in [20].

An other type of partial differential equations, which can be treated as generalisations of CLAIRAUT's equation (1.1), was considered in [21]. Namely, if in equation (1.1) the function y is replaced by a function $u(x, y)$, the derivative y' by the expression $F(x, y)\frac{\partial u}{\partial x} + G(x, y)\frac{\partial u}{\partial y}$, and the variable x by $X(x, y)$ so that $FX_x + GX_y = 1$, we arrive at the equation

$$u = X(x, y)\left(F(x, y)\frac{\partial u}{\partial x} + G(x, y)\frac{\partial u}{\partial y}\right) + f\left(F(x, y)\frac{\partial u}{\partial x} + G(x, y)\frac{\partial u}{\partial y}\right), \tag{5.2}$$

whose general solution is

$$u = A(x, y)\, X(x, y) + f(A(x, y)),$$

where $FA_x + GA_y = 0$. Similarly, if $y = S(x)$ is the singular solution of the equation (1.1), then the singular solution of (5.2) is $u = S(X(x, y))$. For example, the equation

$$u = (xu_x + yu_y)\log x + (xu_x + yu_y)^2,$$

has the general solution

$$u = A\left(\frac{x}{y}\right)\log x + A\left(\frac{x}{y}\right)^2 \qquad (A \text{ arbitrary function})$$

and the singular solution $u = -\frac{\log^2 x}{4}$.

Clearly, higher order equations can be generalised in an analogous manner.

6. Difference equations. Difference equations of CLAIRAUT's type were considered by KLAMKIN in the mentioned paper [10]. He proved the result:

Any sequence of the form $x_n = \sum_{k=0}^{m-1} a_k \binom{n}{k}$, where $a_0, \ldots, a_{m-1}$ are arbitrary constants such that $F(a_0, \ldots, a_{m-1}) = 0$, is a solution of the difference equation

$$F(z_0, z_1, \ldots, z_{m-1}) = 0,$$

where

$$z_r = \Delta^r x_n - n\,\Delta^{r+1} x_n + \frac{1}{2} n(n+1)\,\Delta^{r+2} x_n - \cdots$$

$$+ \frac{(-1)^{m-r-1}}{(m-r-1)!} n(n+1)\cdots(n+m-r-2)\,\Delta^{m-1} x_n.$$

The equation

$$y(x) = x\,\Delta y(x) + f(\Delta y(x)), \tag{6.1}$$

which is directly analogous to (1.1) is considered in book **[22]**, p. 183. Its general solution

$$y(x) = xA(x) + f(A(x)),$$

where A is an arbitrary l-periodic function, is determined. It is also mentioned that the equation (6.1) has other solutions, which can be found by solving the equation

$$\frac{f(z+\Delta z) - f(z)}{\Delta z} = -(x+1).$$

A special case of (6.1), namely the equation

$$x_n = n\,\Delta x_n + (\Delta x_n)^2 \tag{6.2}$$

is considered in book **[23]**, p. 197. The following solutions are found:

$$x_n = Cn + C^2; \qquad x_n = -\frac{1}{4} n^2 + \frac{1}{16} + 4\,C^2 + C(-1)^n.$$

The equation (6.2) is analogous to CLAIRAUT's equation $y = xy' + (y')^2$, which has general solution $y = Cx + C^2$ and singular solution $y = -\frac{1}{4}x^2$.

This example shows that though there is complete analogy between the general solutions of the corresponding CLAIRAUT's differential and difference equations, this analogy breaks down in the case of singular solutions. In fact, the equation (6.2) has a family of singular solutions.

In paper **[24]** it is shown that the equation

$$x_n = \sum_{k=1}^{m} \frac{(-1)^{k-1}}{k!} n(n+1)\cdots(n+k-1)\,\Delta^k x_n + f(\Delta^m x_n), \tag{6.3}$$

which is CLAIRAUT's m-th order equation, has the general solution

$$x_n = \sum_{k=1}^{m} \frac{1}{k!} C_k\, n^{(k)} + f(C_m),$$

where $n^{(k)} = n(n-1)\cdots(n-k+1)$. This is in accordance with KLAMKIN's result from [10]. It is also shown that the singular solution can be obtained by solving the equation

$$\frac{f(\Delta^m x_{n+1}) - f(\Delta^m x_n)}{\Delta^{m+1} x_n} = \frac{(-1)^m}{m!} n(n+1)\cdots(n+m).$$

In particular, for the equation

$$x_n = n\,\Delta x_n - \frac{1}{2} n(n+1)\,\Delta^2 x_n + (\Delta^2 x_n)^2 \tag{6.4}$$

the singular solution was explicitly found to be

$$x_n = \frac{1}{48} n^4 - \frac{1}{12} n^2 + \frac{1}{64} + C_1 n + 16\,C_2^2 + C_2(-1)^n.$$

The examples of equtions (6.2) and (6.4) suggest that the singular solution of (6.3) contains m arbitrary constants (the same number as the general solution).

An attempt was also made in [24] to give a geometric interpretation of the relation between singular and general solutions of CLAIRAUT's difference equations.

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VARIJACIJE I GENERALIZACIJE CLAIRAUTOVIH JEDNAČINA

D. S. Mitrinović i *J. D. Kečkić*

Clairautova diferencijalna jednačina (1.1) čije je opšte rešenje (1.2), a singularno (1.3), više puta je uopštena i to uglavnom u sledećim pravcima: određivanje najopštije jednačine prvog reda koja ima osobinu Clairautove jednačine, određivanje diferencijalnih jednačina višeg reda sa osobinom Clairautove jednačine, određivanje parcijalnih diferencijalnih jednačina čiji potpuni integrali imaju osobinu Clairautove jednačine, ispitivanje singularnih rešenja i određivanje diferencnih jednačina sa analognim osobinama. U ovom radu dat je istorijski pregled tih uopštenja, pri čemu je svakom od navedenih pravaca posvećen po jedan odeljak.